Variational methods for topological patterns arising in physics

Report of Contributions

https://indico.math.cnrs.fr/e/15095

Contribution ID: **1** Type: **not specified**

Solitons and solitonic vortices as mountain passes for the Ginzburg-Landau energy in 2D strips and 3D cylinders

Tuesday, November 25, 2025 2:30 PM (45 minutes)

We study critical points of the Ginzburg-Landau energy on 2D strips and 3D cylinders. In relation with recent experiments on fermionic and bosonic strips, we prove that there is a critical width of the strip under which the minimizer in some suitable space is the soliton while above it the minimizer is solitonic vortex (aka a vortex which behaves like a soliton in the transverse direction). We manage to go further and characterize them as a mountain pass solutions. In 3d, we present generalizations and open questions. Joint work with Amandine Aftalion.

Presenter: NGUYEN, Luc (Oxford)

Contribution ID: **2** Type: **not specified**

An approach to topological singularities through Mumford-Shah type functionals

Tuesday, November 25, 2025 3:20 PM (45 minutes)

We will present variational approaches to the analysis of topological singularities in the plane, starting from the - nowadays - classical Ginzburg-Landau (GL) model and core-radius (CR) approach. We will introduce a third approach inspired by the Mumford-Shah functional used in the context of image segmentation. Within our framework, the order parameter is an SBV map taking values in the unit sphere of the plane; the bulk energy is the squared L2 norm of the approximate gradient whereas the penalization term is given by the length of the jump set, scaled by a small parameter. After providing a notion of Jacobian determinant for SBV maps, we show that at any logarithmic scale our functional is "variationally equivalent" to the "standard" (CR) and (GL) models. Joint work with Vito Crismale, Nicolas Van Goethem and Riccardo Scala.

Presenter: DE LUCA, Lucia (Rome)

Contribution ID: **3** Type: **not specified**

On a nonlinear Schrödinger equation: uniqueness, non-degeneracy and applications to energy minimization

Tuesday, November 25, 2025 4:40 PM (45 minutes)

This talk will focus on qualitative properties of normalized ground states for a nonlinear Schrödinger equation with double-power nonlinearity. These ground states are characterized as energy minimizers under a fixed L^2-norm contraint. I will present recent results concerning their existence, and I will discuss their uniqueness in certain regimes of parameters. Joint work with Mathieu Lewin.

Presenter: ROTA NODARI, Simona (Nice)

Contribution ID: **4** Type: **not specified**

Domain wall dynamics in notched ferromagnetic nanowires

Wednesday, November 26, 2025 9:20 AM (45 minutes)

We study the dynamics of domain walls in a notched ferro-magnetic nanowire. The model used is the Landau–Lifschitz equation in dimension 1, with a weight representing the notch. We highlight the pinning properties of notches, and the depinning properties of the applied magnetic field. In particular, we establish that when the applied field tends to push the wall far away from the notch, pinning effects are negligible.

Presenter: CARBOU, Gilles (Pau)

Contribution ID: **5** Type: **not specified**

Stabilization of walls in notched ferromagnetic nanowires

Wednesday, November 26, 2025 10:40 AM (45 minutes)

We present a one-dimensional model of ferromagnetic nanowire featuring notches. We prove the existence of stable wall profiles even under a small applied magnetic field with the walls localized in notches. Moreover, in order to illustrate domain-wall depinning by an applied magnetic field, we prove the non-existence of stationary wall profiles in the presence of a large applied magnetic field.

Presenter: SANCHEZ, David (Toulouse)

Contribution ID: **6** Type: **not specified**

Existence and uniqueness of a domain wall for notched ferromagnetic nanowires

Wednesday, November 26, 2025 11:30 AM (45 minutes)

In this talk, we study a ferromagnetic nanowire with a defect, represented as a single, unimodal notch. Using a mountain-path argument, we establish the existence and uniqueness of a critical point for the ferromagnetic energy associated with this model. This critical point corresponds to a topological solution (a single domain wall) localized in the vicinity of the notch. This work allows the results [Carbou 2020, Carbou&Sanchez 2018, Ignat 2022] to be reinterpreted using the framework of the calculus of variations.
This is a joint work with Raphaël Côte, Guillaume Ferrière, Ludovic Godard-Cadillac and Yannick Privat.

Presenter: COURTES, Clémentine (Strasbourg)

Contribution ID: **7** Type: **not specified**

The conformal limit for bimerons in easy-plane chiral magnets

Wednesday, November 26, 2025 2:30 PM (45 minutes)

We study minimizers of a reduced micromagnetic energy functional under prescribed unit topological degree. This model arises in thin ferromagnetic films with Dzyaloshinskii-Moriya interaction and easy-plane anisotropy, where these minimizers represent bimeron configurations. We prove their existence, and describe them precisely as perturbations of specific Möbius maps: we establish in particular that they are localized at a logarithmic scale. The proof follows a strategy introduced by Bernand-Mantel, Muratov and Simon(Arch. Ration. Mech. Anal., 2021) for a similar model with easy-axis anisotropy, but requires several adaptations to deal with the less coercive easy-plane anisotropy and different symmetry properties.
This is a joint work with R. Ignat and X. Lamy.

Presenter: DENG, Bin (Toulouse)

Contribution ID: **8** Type: **not specified**

Korn and Poincaré-Korn inequalities: A different perspective

Wednesday, November 26, 2025 3:20 PM (45 minutes)

Korn's inequality and its variants are essential tools in the mathematical analysis of both linear and nonlinear elasticity. They play a central role in establishing existence and regularity results for partial differential equations involving symmetric gradients. In this talk, I will present a conceptually simple derivation of the first and second Korn inequalities for general exponents $1 < p < \infty$, applicable to a wide class of domains, including Lipschitz and extension domains. Our approach bypasses the traditional reliance on singular integral estimates and intricate geometric arguments, instead relying on the classical and q-Riesz representation theorems. In the case $p = 2$, the argument becomes especially transparent, requiring only basic Hilbert space methods and Weyl's lemma. I will also discuss associated Poincaré–Korn inequalities in both bounded and unbounded domains, which remain valid even in the limiting case $p = 1$. These inequalities not only ensure the coercivity of variational problems in elasticity but also serve as a key preliminary step in the proof of the first Korn inequality.

Presenter: DI FRATTA, Giovanni (Naples)

Contribution ID: **9** Type: **not specified**

TBA

Wednesday, November 26, 2025 4:40 PM (45 minutes)

TBA

Presenter: GARRONI, Adriana (Rome)

Contribution ID: **10** Type: **not specified**

A variational model for two-dimensional ferronematics - Part I

Thursday, November 27, 2025 9:20 AM (45 minutes)

This talk is the first of two parts, presented jointly with Federico Luigi Dipasquale. Ferronematics are composite materials characterised by the coupling between magnetic particles and nematic liquid crystals. In these talks, we will present some results on a two-dimensional model for ferronematics in confined geometries. The model is based on the coupling between a polar order parameter –the magnetisation vector, which describes the magnetic inclusions - and a nonpolar one - the Landau-de Gennes Q-tensor, which describes the liquid crystal matrix. In this first talk, we will introduce the model and discuss the qualitative behaviour of free-energy minimisers in some asymptotic regime of parameters, where both point and line singularities appear. This talk is based on a joint work with G.Canevari, A. Majumdar and Y. Wang.

Presenter: STROFFOLINI, Bianca (Naples)

Contribution ID: **11** Type: **not specified**

Ferronematics: Asymptotics for critical points

Thursday, November 27, 2025 10:40 AM (45 minutes)

We consider a variational model for ferronematics — composite materials formed by dispersing magnetic nanoparticles into a liquid crystal matrix. The model features two coupled order parameters: a Landau-de Gennes Q-tensor for the liquid crystal component and a magnetisation vector field M, both of them governed by a Ginzburg-Landau-type energy. The energy includes a singular coupling term favouring alignment between Q and M. We report on some recent results on the asymptotic behaviour of (not necessarily minimizing) critical points as a small parameter eps tends to zero. Our main results show that the energy concentrates along distinct singular sets: the (rescaled) energy density for the Q-component concentrates, to leading order, on a finite number of singular points, while the energy density for the M-component concentrates along a one-dimensional rectifiable set. Moreover, we will see that the curvature of the singular set for the M-component (technically, the first variation of the associated varifold) is concentrated on a finite number of points, i.e. the singular set for the Q-component.
Joint work with G.Canevari (University of Verona) and B. Stroffolini (University of Naples "Federico II").

Presenter: DIPASQUALE, Federico (Naples)

Contribution ID: **12** Type: **not specified**

Convergent finite element methods for the Ericksen model of nematic liquid crystals

Thursday, November 27, 2025 11:30 AM (45 minutes)

The Ericksen model describes nematic liquid crystals (LCs) in terms of a unit-length vector field (director) and a scalar function (degree of orientation). Compared to the classical Oseen-Frank model, it allows for the description of a larger class of defects. Equilibrium states of the LC are given by admissible pairs that minimize an energy functional, which consists of the sum of Oseen-Frank-like energies and a double well potential. The resulting Euler-Lagrange equations are degenerate for the director, which poses serious difficulties to formulate mathematically sound algorithms for their approximation. We propose a simple but novel finite element approximation of the problem that does not employ a projection to impose the unit-length constraint on the director and thus circumvents the use of weakly acute meshes, quite restrictive in 3D. We show stability and Gamma-convergence properties of the new method in the presence of defects. We also discuss an effective nested gradient flow algorithm for computing minimizers that controls the violation of the unit-length constraint. We present several simulations in 2D and 3D that document the performance of the proposed scheme and its ability to capture quite intriguing defects. This is joint work with Ricardo H. Nochetto (University of Maryland) and Shuo Yang (BIMSA).

Presenter: RUGGERI, Michele (Bologna)

Contribution ID: **13** Type: **not specified**

Some progress on the Aviles-Giga conjecture

Thursday, November 27, 2025 2:30 PM (45 minutes)

The Aviles-Giga energy is a phase transition model related to liquid crystals, micromagnetics and elasticity. Sharp interface limits of bounded energy are weak solutions of the 2D eikonal equation: unit vector fields m with zero divergence (in the sense of distributions). Partial information about the limit energy cost of a given solution m is encoded in a family of signed measures called entropy productions. It is conjectured that these measures are concentrated on the 1-rectifiable jump set of m, as they do if m has bounded variation (BV). In a joint work with Elio Marconi, we prove this concentration property under an additional mild regularity assumption, going well beyond the BV setting, and leaving only a borderline case open.

Presenter: LAMY, Xavier (Toulouse)

Contribution ID: **14** Type: **not specified**

Aviles-Giga type functionals calibrated with the help of currents

Thursday, November 27, 2025 3:20 PM (45 minutes)

For vector fields in a two-dimensional domain, consider a Modica-Mortola (or Allen-Cahn) type functional. We do not make any specific assumptions on the wells of the potential function (so there may be multiple single-point wells or one or several more complex wells), but we do assume that the divergence of the vector fields is quite strongly penalised or even vanishes identically. This then gives rise to a situation similar to the Aviles-Giga functionals.

As for other Modica-Mortola type problems (or as for the classical Aviles-Giga problem), when we let the relevant parameter tend to 0, we expect a limit that takes values in the wells of the potential function, but there can be transitions between different values. We want to find out how much energy is required for such a transition.

Our strategy is to find suitable functions, called calibrations, which measure the energy of a transition layer. This is not a new idea, but we take it in a somewhat different direction. When asking for the optimal calibration for a given potential function, we are led to a variational problem involving the L-infinty norm of the gradient. Such problems are difficult to study, but using recent ideas of Katzourakis and myself, we can encapsulate the essential information in a geometric variational problem. Solving the latter can still be difficult, but we finally obtain an answer at least in some cases.

This is joint work with Radu Ignat (Toulouse).

Presenter: MOSER, Roger (Bath)

Contribution ID: **15** Type: **not specified**

The binormal flow with Lipschitz non-closed curves data

Thursday, November 27, 2025 4:40 PM (45 minutes)

The binormal flow is a geometric flow of curves in R^3, that models vortex filament dynamics in fluids and superfluids, and is also related to the continuous classical Heisenberg ferromagnet equation. The result presented in this talk is the construction of weak solutions for non-closed Lipschitz curves data, and it extends the result of Bob Jerrard and Didier Smets obtained in 2015 for closed curves. This allows to consider general data including the known examples of strong solutions that generate singularities in finite time. The descriptions of the latter are precise, based on the Hasimoto transform that links the binormal flow to the 1D cubic Schrödinger equation, and on the explicit expression of the self-similar solutions. In the present general low regularity context this approach is no longer available and geometric measure theory tools are used instead. More precisely, we introduce a notion of renormalized length for non-closed curves and work in the framework of locally integral currents. This is a joint work with Bob Jerrard and Didier Smets.

Presenter: BANICA, Valeria (Paris)

Contribution ID: **16** Type: **not specified**

Energy of semi-coherent interfaces

Friday, November 28, 2025 9:20 AM (45 minutes)

In this talk I will discuss models for analyzing inter-crystalline
boundaries that arise from differences in atomic spacing. In the case of
semi-coherent interfaces, where the misfit between crystal lattices is small, the interfaces can be resolved into sequences of edge dislocations,
leading to an interfacial energy that exhibits Read–Shockley-type superlinear scaling as a function of the misfit.

Presenter: SPADARO, Emanuele (Rome)

Contribution ID: **17** Type: **not specified**

Phase-field approximation of sharp-interface energies accounting for lattice symmetry

Friday, November 28, 2025 10:40 AM (45 minutes)

The talk concerns a phase field approximation for sharp interface energies, defined on partitions, as appropriate for modeling grain boundaries in polycrystals. The label takes value in O(d)/G, where G is the point group of a lattice. The limiting surface energy behaves for small angles as s|log s|, according to the Read and Shockley law. These functionals can be used for image reconstruction of grain boundaries. Joint work with S. Conti (HCM Bonn), A. Garroni, A. Malusa (Sapienza).

Presenter: CRISMALE, Vito (Rome)

Contribution ID: **18** Type: **not specified**

A ”not so strange” term in the homogenisation of a problem with Robin boundary conditions

Friday, November 28, 2025 11:30 AM (45 minutes)

We consider Laplace’s equation in a periodically perforated domain, with Robin boundary conditions on the holes and a Robin coefficient inversely proportional to the total surface area of the holes. We show that, in a critical regime, the homogenised equation contains an additional zero-order term, which is defined in terms of a suitable eigenvalue problem and depends nonlinearly on the Robin coefficient. As the latter tends to infinity, the additional term converges to the capacitary “strange term” found by Cioranescu and Murat in the homogenisation of a problem with Dirichlet boundary conditions. This talk is based on joint work with K. Cherednichenko (University of Bath) and A. Zarnescu (BCAM, Bilbao, and “Simion Stoilow” Institute of the Romanian Academy).

Presenter: CANEVARI, Giacomo (Verona)