Chirality-enabled knot solitons and knotted vortices as topological metaatoms

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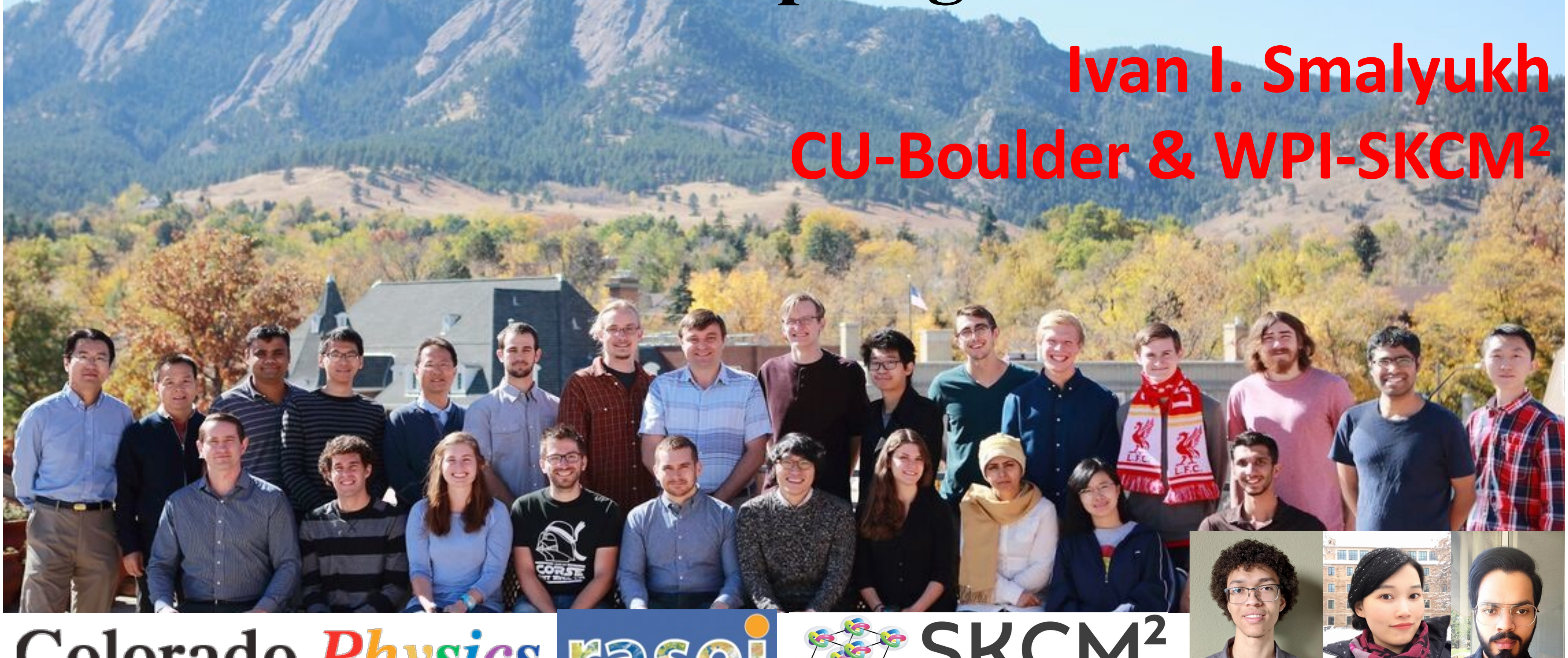

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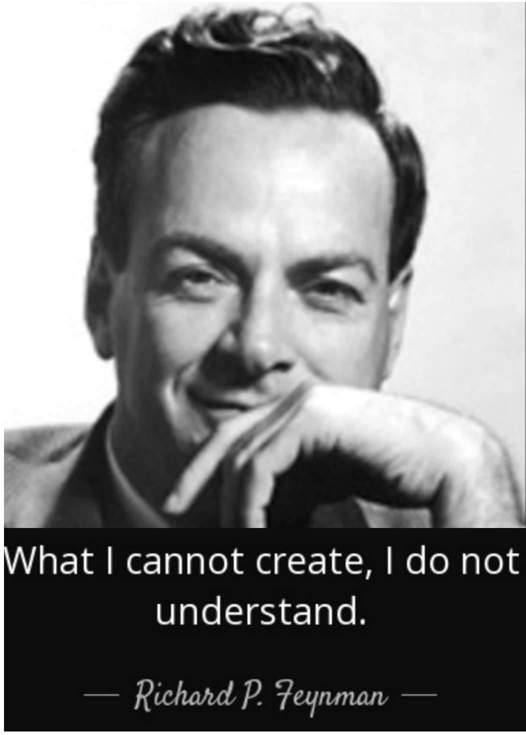
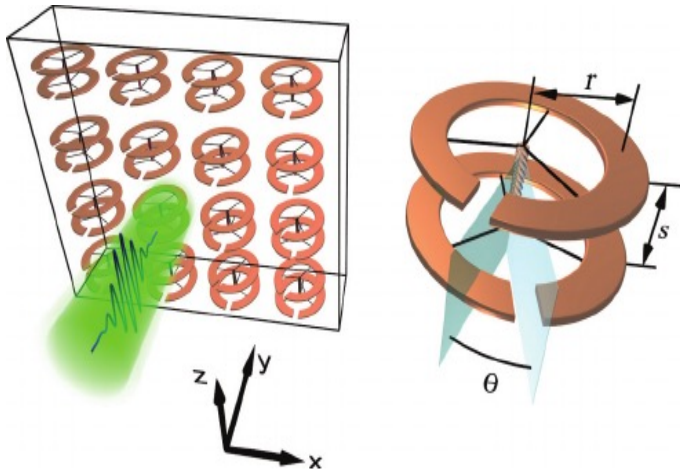
WPI HIROSHIMA UNIVERSITY

Darian Hall, Cuiling Meng, Amit Bhardwaj

Universality & analogies

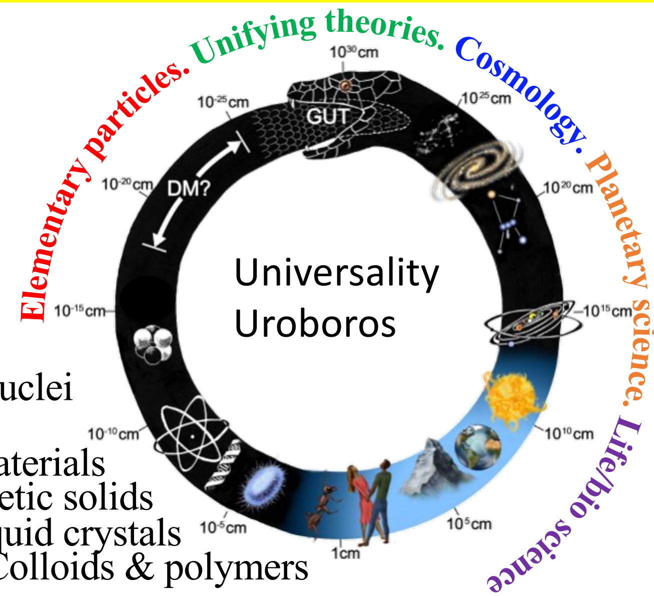


Metamaterials from designed metaatoms



Unusual properties like negative refraction

Phys. Rev. B 87, 235126 (2013)



Atomic nuclei

Quantum materials

Magnetic solids

Liquid crystals

Colloids & polymers

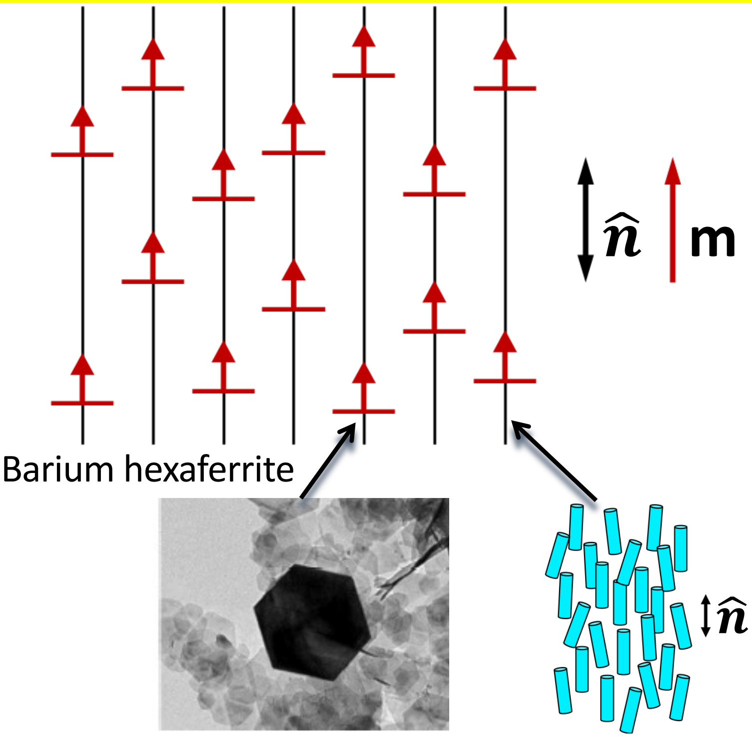
→Vortex knots as metaatoms (or subatomic particles)

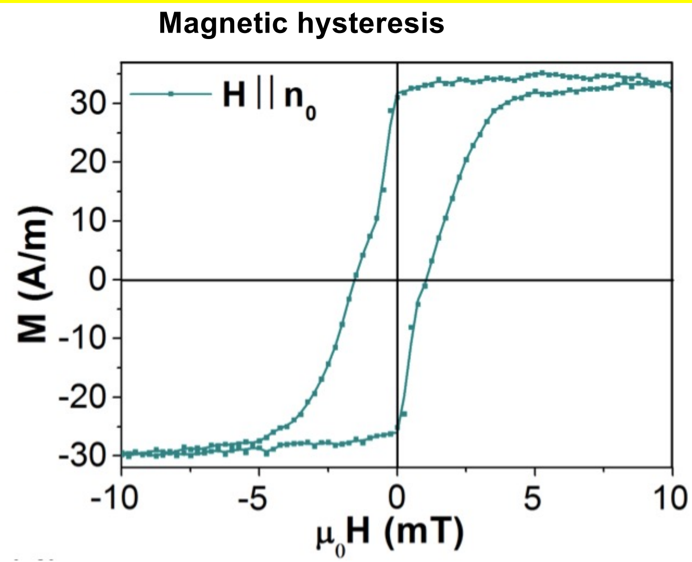
→Crystals of knots & giant electro-striction

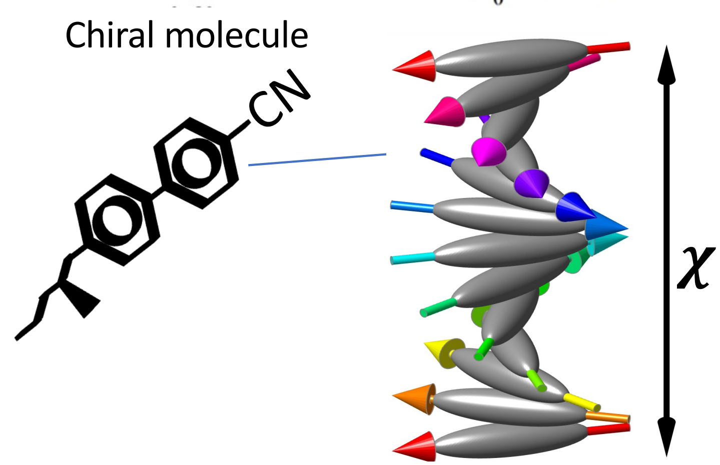
→Fusion, fission & hopping of knots

→Steering & knotting light by vortex analogs of cosmic strings

Colloidal magnetic chiral liquid crystals (LCs)







→Ferromagnetic nanoplates dispersed in nematic LC
→Magnetization field $\boldsymbol{m}(\boldsymbol{r})$ parallel to $\boldsymbol{n}(\boldsymbol{r})$
→Polar switching by weak fields $\sim$10G

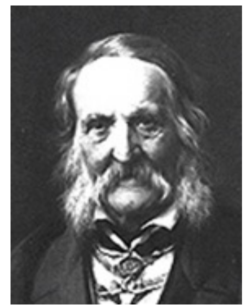
Mertelj, et al. *Nature* **504**, 237 (2013); Q. Zhang, et al. *Phys. Rev. Lett.* **115**, 097802 (2015).

Why is symmetry important?

Neumann-Curie Principle

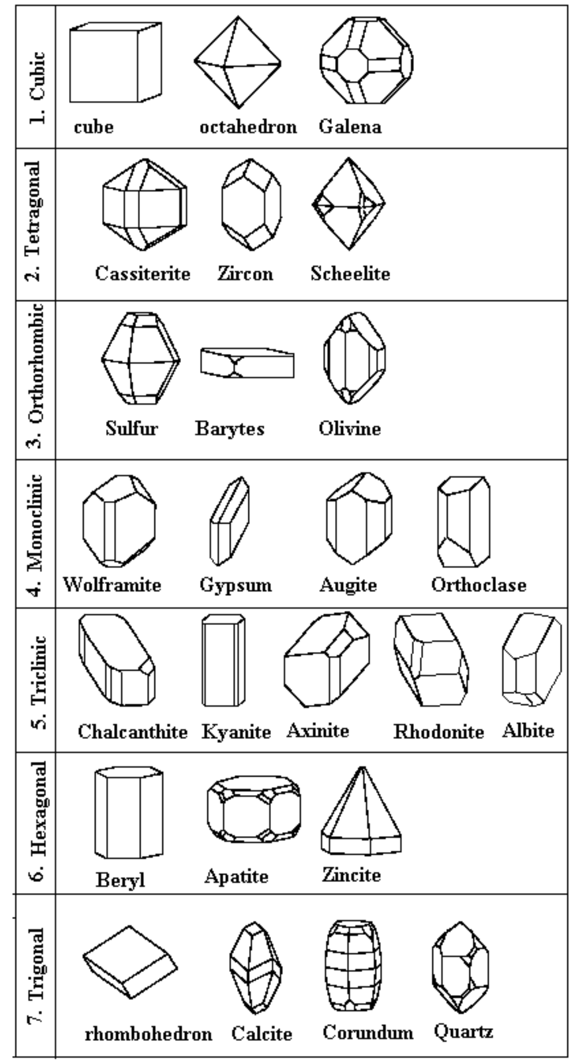
- Neumann: "In its physical properties, a material exhibits the same kind of symmetry as its crystallographic shape."
- Curie: The physical properties of a system are invariant to its symmetry.

Franz Neumann
1798-1895



Pierre Curie
1859-1906

Crystal form of the seven crystal systems



Curie’s Corollary:

C’est la dissymétrie qui crée la phénomène.

“Phenomena are created by a reduction in symmetry.” (1894)

Physical/chemical properties owe their very existence to the *absence* of certain symmetry operations.

Role of chirality in LCs, colloids & solid-state magnets

→Hamiltonian for **m**(**r**)-distortions in fluid chiral LC colloidal magnets

$$F_{CFLCC} = \int d\mathbf{r} \left\{ \frac{K_{11}}{2}(\nabla \cdot \mathbf{m})^2 + \frac{K_{22}}{2}[\mathbf{m} \cdot (\nabla \times \mathbf{m})]^2 + \frac{K_{33}}{2}[\mathbf{m} \times (\nabla \times \mathbf{m})]^2 + q_0 K_{22} \mathbf{m} \cdot (\nabla \times \mathbf{m}) \right\}$$

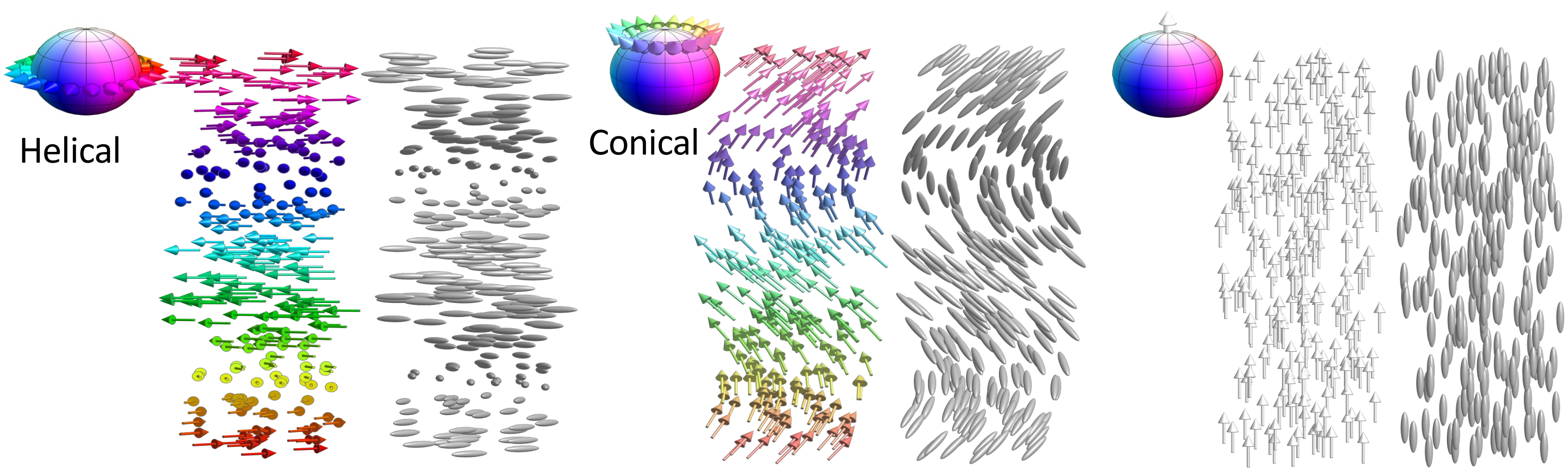
→**Chirality & overcoming Derrick theorem's constraints**

→Additional terms in presence of external fields

$$F_{\text{dielectric}} = -\frac{\varepsilon_0 \Delta\varepsilon}{2} \int_\Omega d^3\boldsymbol{r} (\boldsymbol{m} \cdot \boldsymbol{E})^2 . \qquad F_{\text{magnetic}} = -\mu_0 M \int_\Omega d^3\boldsymbol{r} (\boldsymbol{H} \cdot \boldsymbol{m}) .$$

→Hamiltonian for chiral magnetic solids

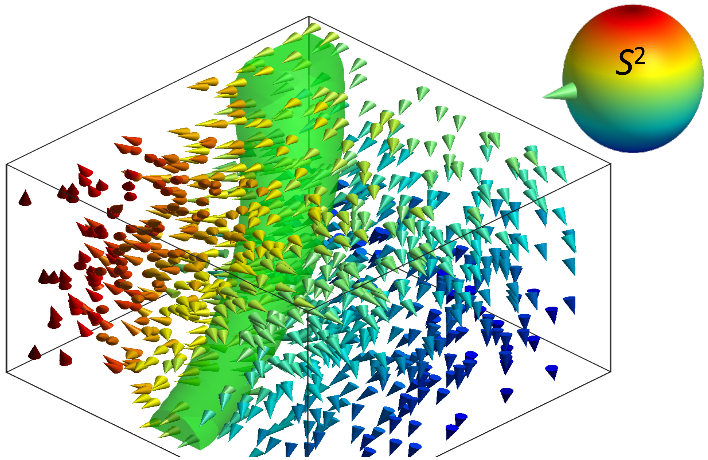
$$H = \int d^3\boldsymbol{r} \left\{ \frac{J}{2} (\nabla \boldsymbol{m})^2 + D\boldsymbol{m} \cdot (\nabla \times \boldsymbol{m}) - \mu_0 M_s (\boldsymbol{H} \cdot \boldsymbol{m}) \right\}$$



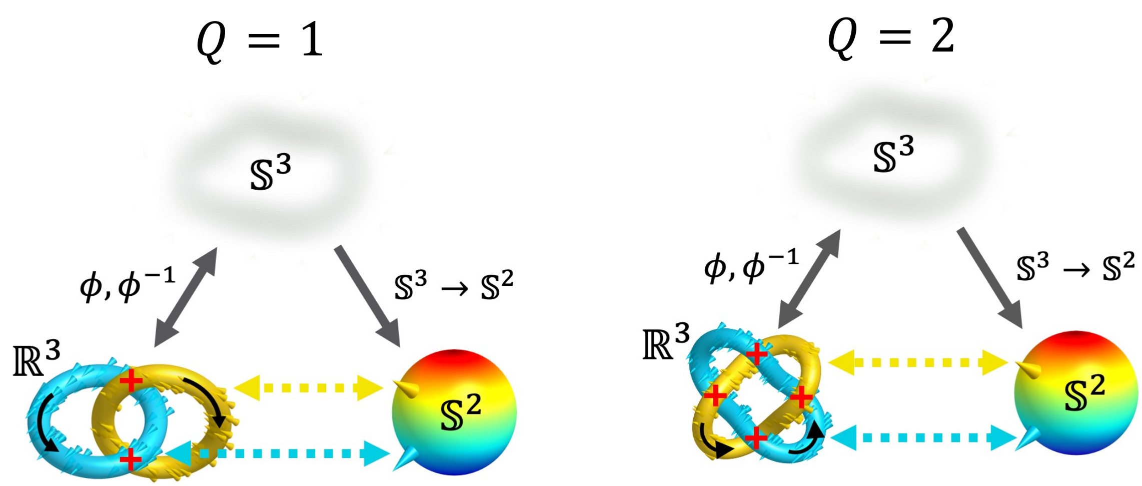
→Equivalent for equal elastic constants K with $J=K/2$, $D=Kq_0$

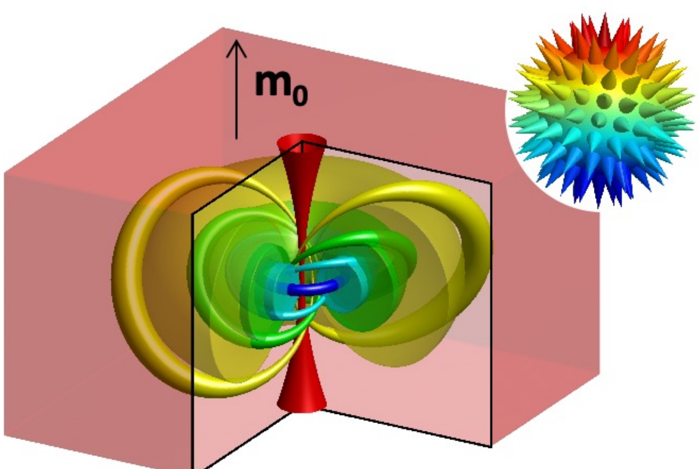
→Mixtures of LC molecules (5CB+CB7CB+CB15)

3D Knot solitons, the hopfions



→$\pi_3(\mathbb{S}^2) = \mathbb{Z} = Q$, the Hopf index
→Q is also the linking number of preimages $Q = \Sigma C/2$

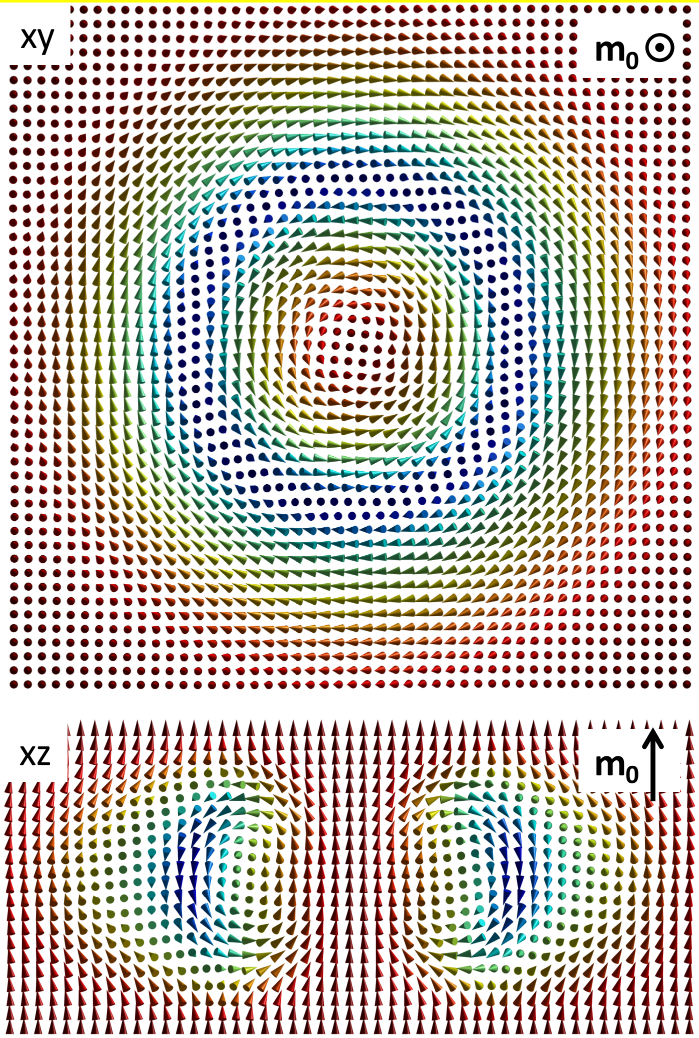


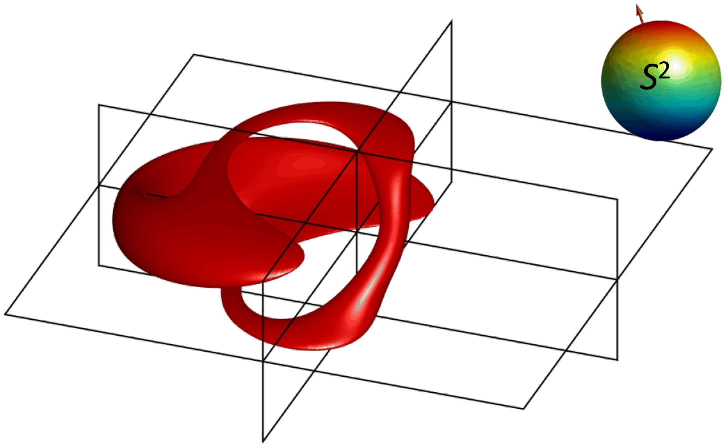


→$\mathbb{R}^3 \cong \mathbb{S}^3$ when far-field uniform

PJ Ackerman and II Smalyukh. *Nat. Mat.* **16**, 426 (2017)
PJ Ackerman and II Smalyukh. *Phys. Rev. X* (2017)
JSB Tai, PJ Ackerman, II Smalyukh, *PNAS U.S.A.* **115**, 921 (2018).

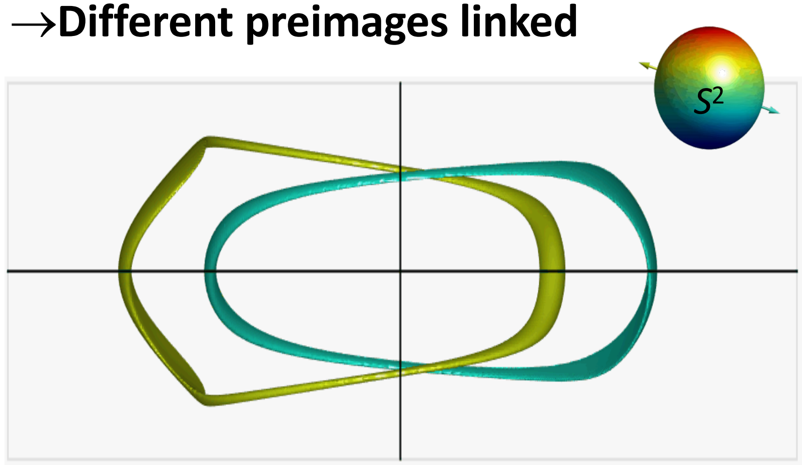
3D Hopf solitons: numerical modeling & analysis





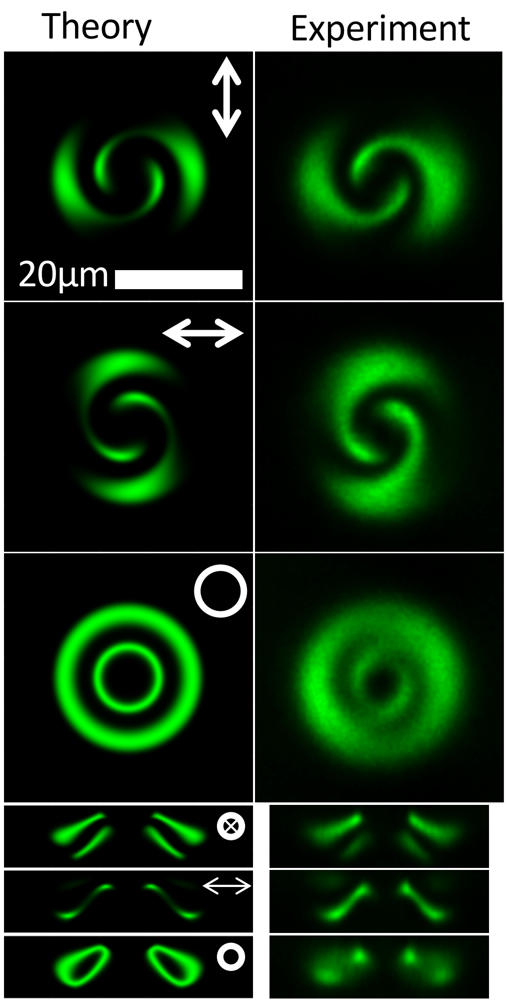
→All preimages - closed loops

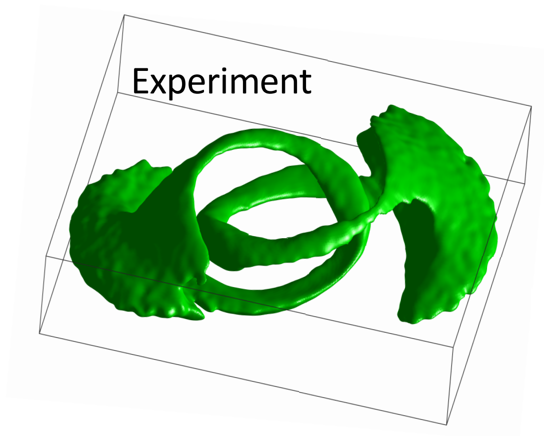
→Different preimages linked

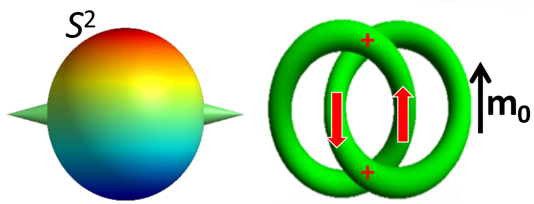


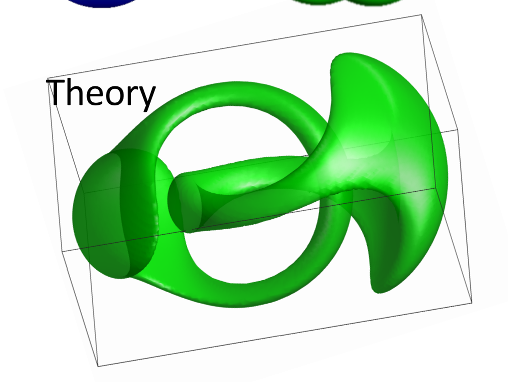
P.J. Ackerman & I.I. Smalyukh. *Nature Mater.* **16**, 426-432 (2017)

Numerical modeling versus experiment

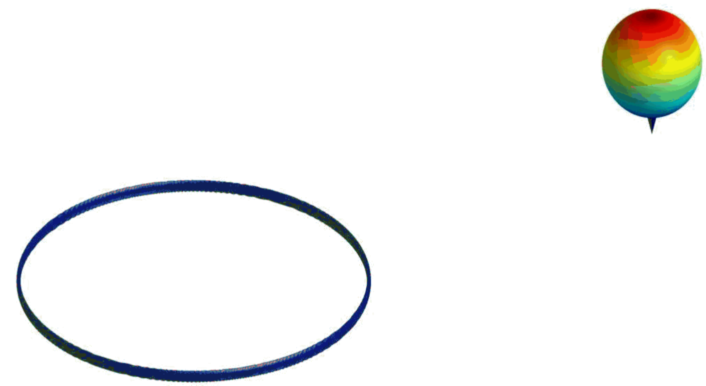






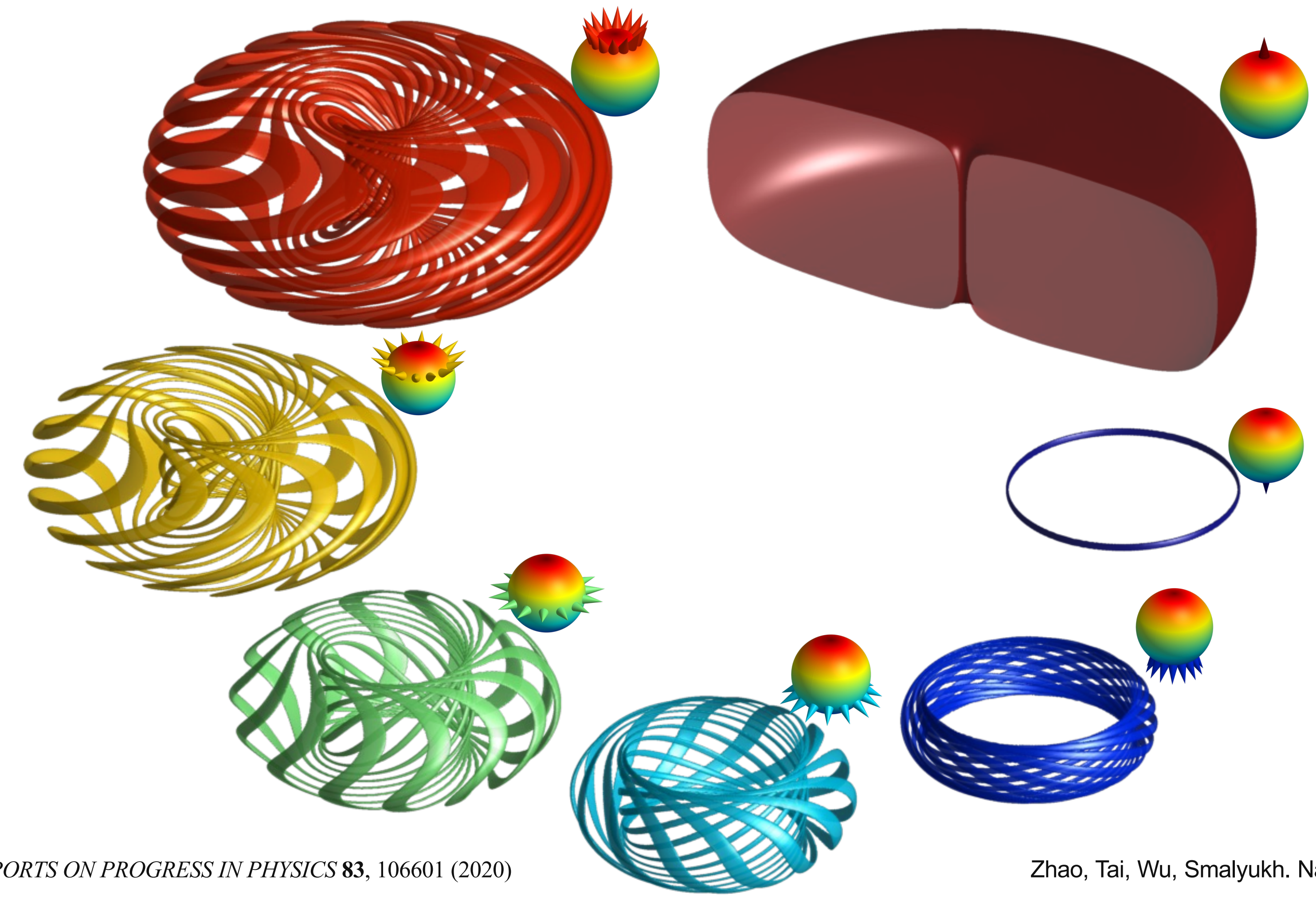


→**Filling localized space with all preimages:**



P.J. Ackerman & I.I. Smalyukh. *Nature Mater.* **16**, 426-432 (2017)

Nested tori of linked preimages within localized hopfion

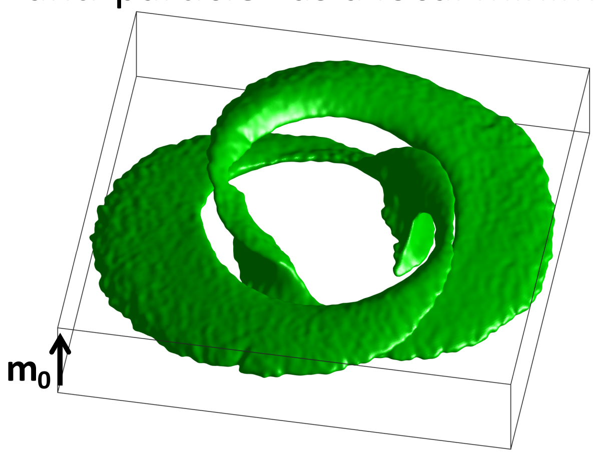


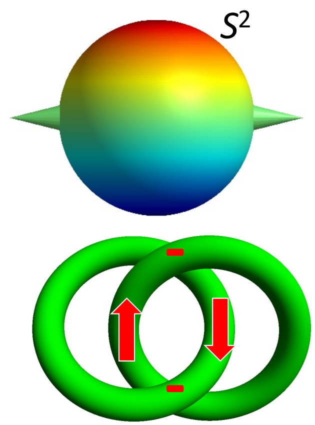
I. Smalyukh. *REPORTS ON PROGRESS IN PHYSICS* **83**, 106601 (2020)

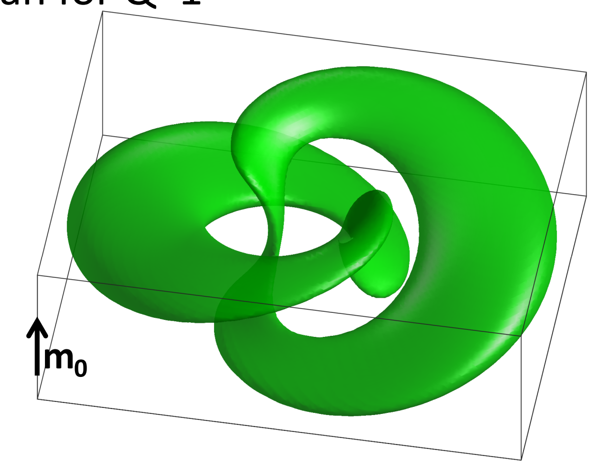
Zhao, Tai, Wu, Smalyukh. Nature Phys. **19**, 451-459 (2023)

Q=-1 solitons & magnetic control of 2D crystals

→Q=-1 "anti-particle" as a local minimum of higher energy than for Q=1

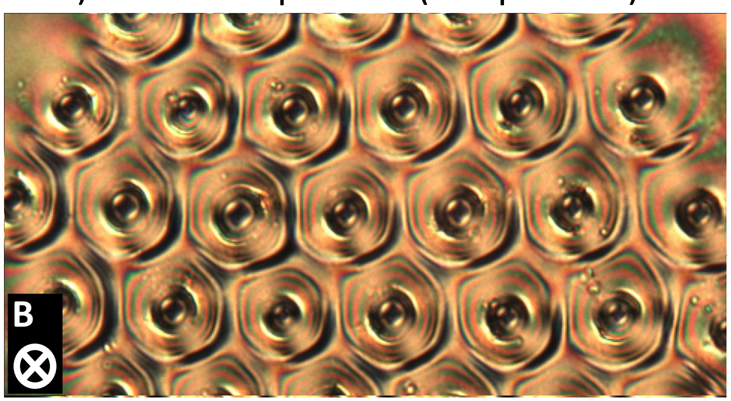


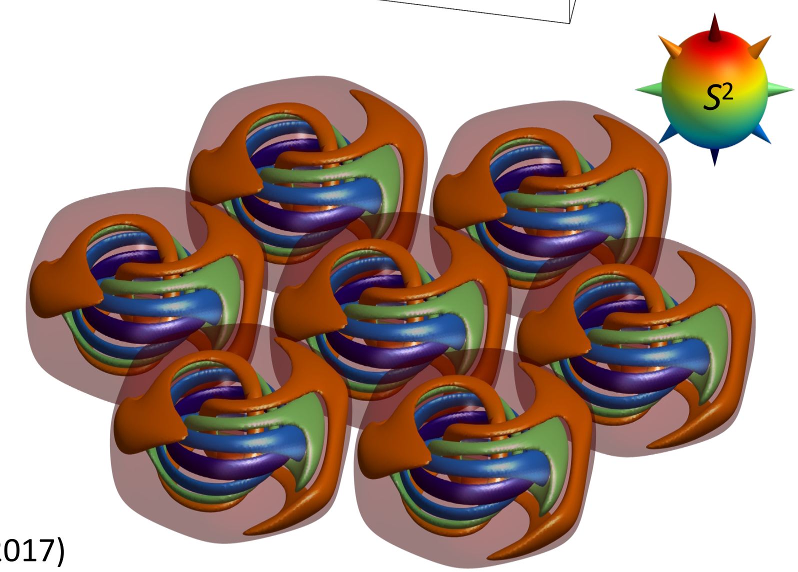




→2D arrays of the 3D solitons

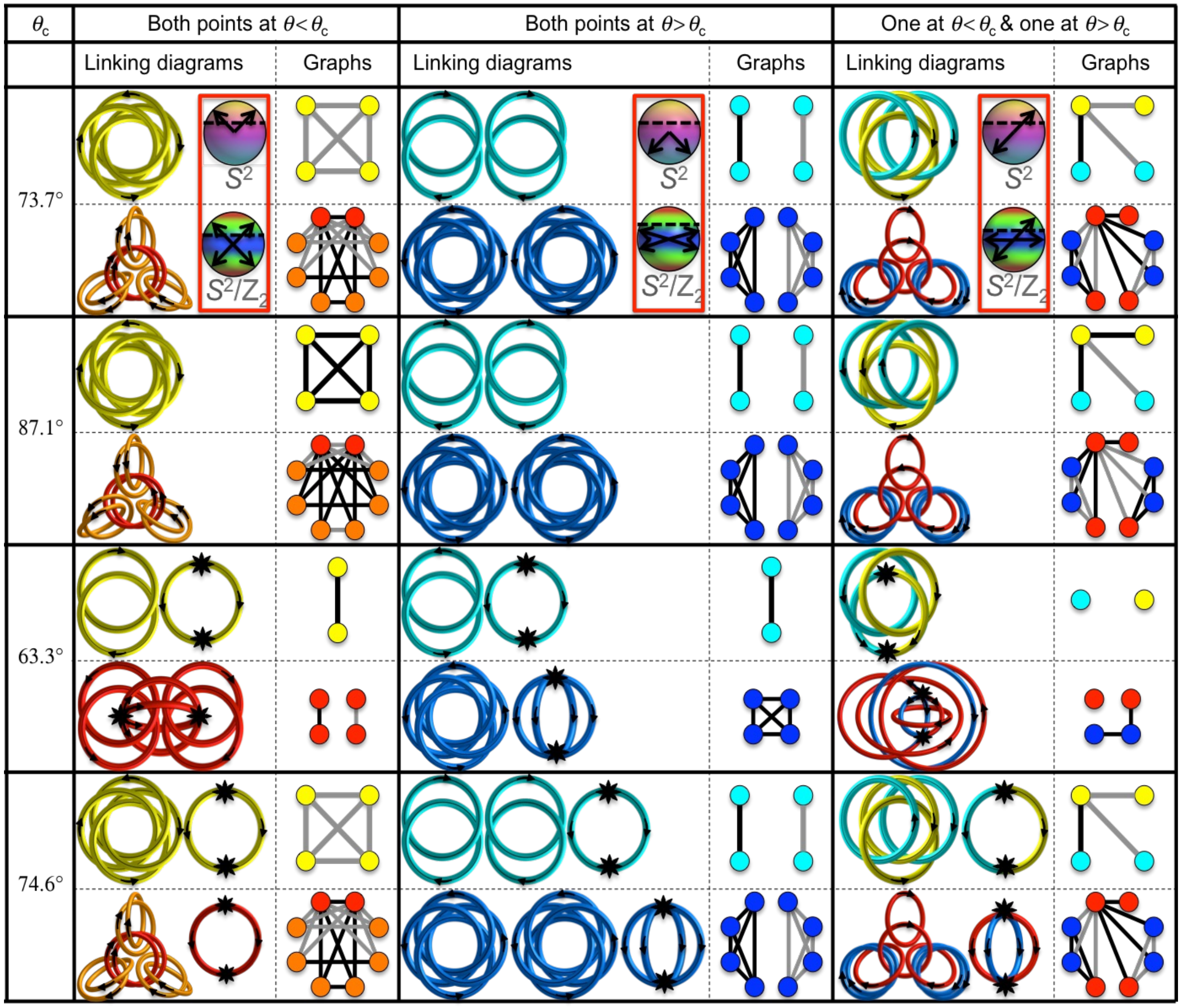
→Shrink (expand) when **B** is parallel (antiparallel) to $\mathbf{M}_0$

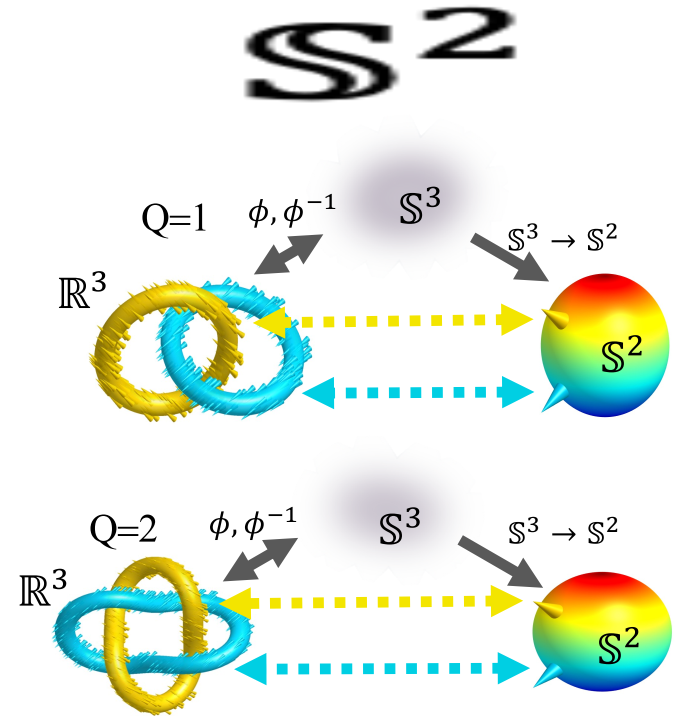




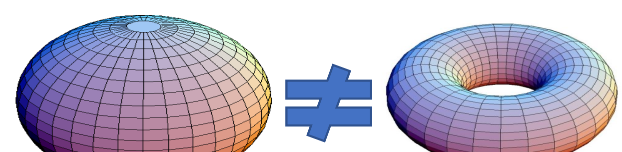
P. J. Ackerman and I. I. Smalyukh. *Phys Rev X* **7**, 011006 (2017)

Beyond elementary Hopf solitons: linking number





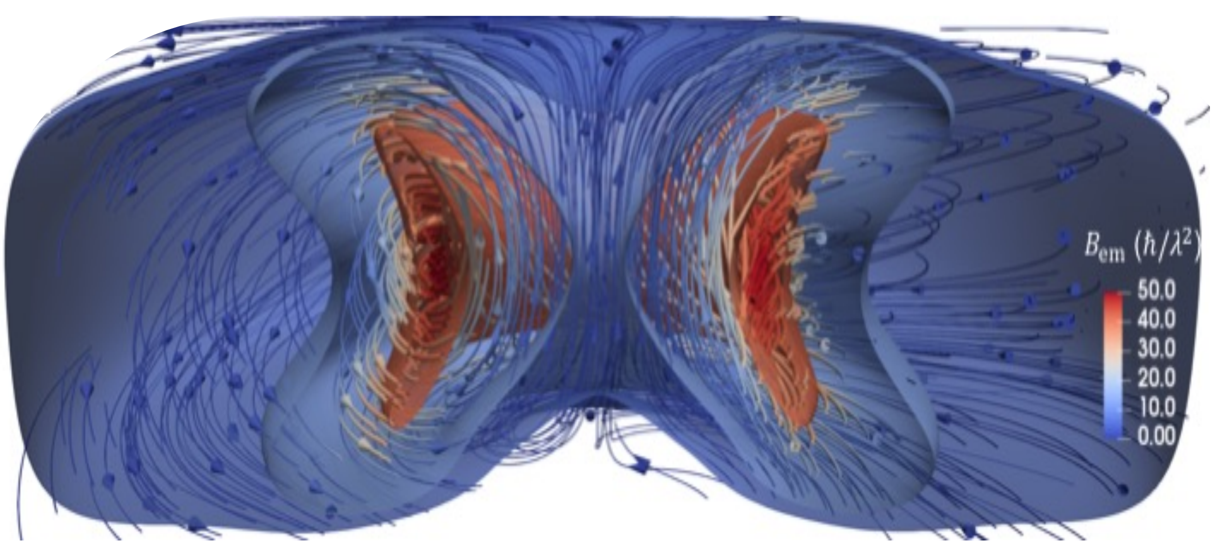
$\pi_3(S^2)=Z$ solitons

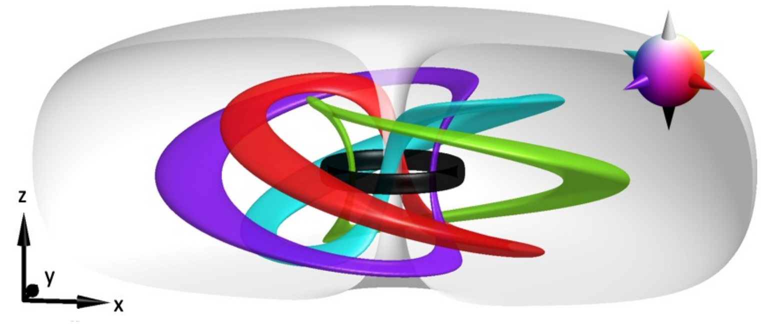
Like 

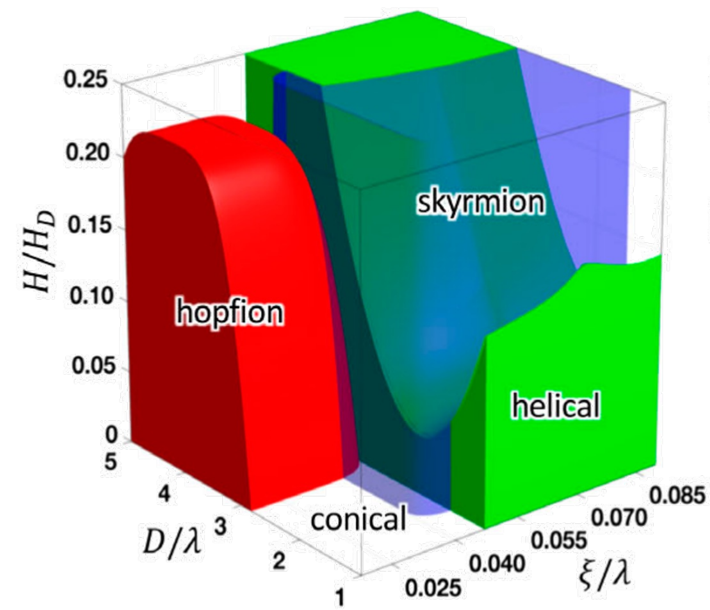
3D Hopf solitons in solid-state chiral magnets

Closed-loop streamlines of emergent field
-> Hopf fibration

$$(\boldsymbol{B}_{\mathrm{em}})_i \equiv \hbar\varepsilon^{ijk}\boldsymbol{m}\cdot(\partial_j\boldsymbol{m}\times\partial_k\boldsymbol{m})/2$$



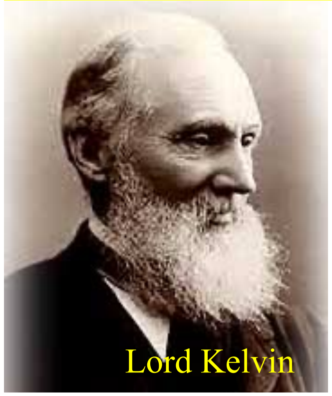




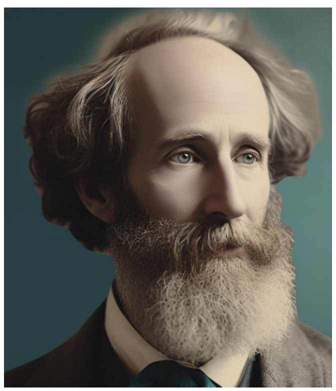
J Tai, I Smalyukh, *Phys. Rev. Lett.* **121**, 187201 (2018).

R. Voinescu, J.-S. B. Tai and I. I. Smalyukh. *Phys Rev Lett* **125**, 057201 (2020)

Knots & chirality in science: shared fascinating history



William Thomson



James Clerk Maxwell

THE PERIODIC TABLE OF ELEMENTS

1 H																	2 He
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba	57-71 -	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	89-103 -	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Uut	114 Uuq	115 Uup	116 Uuh	117 Uus	118 Uuo

57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr

?

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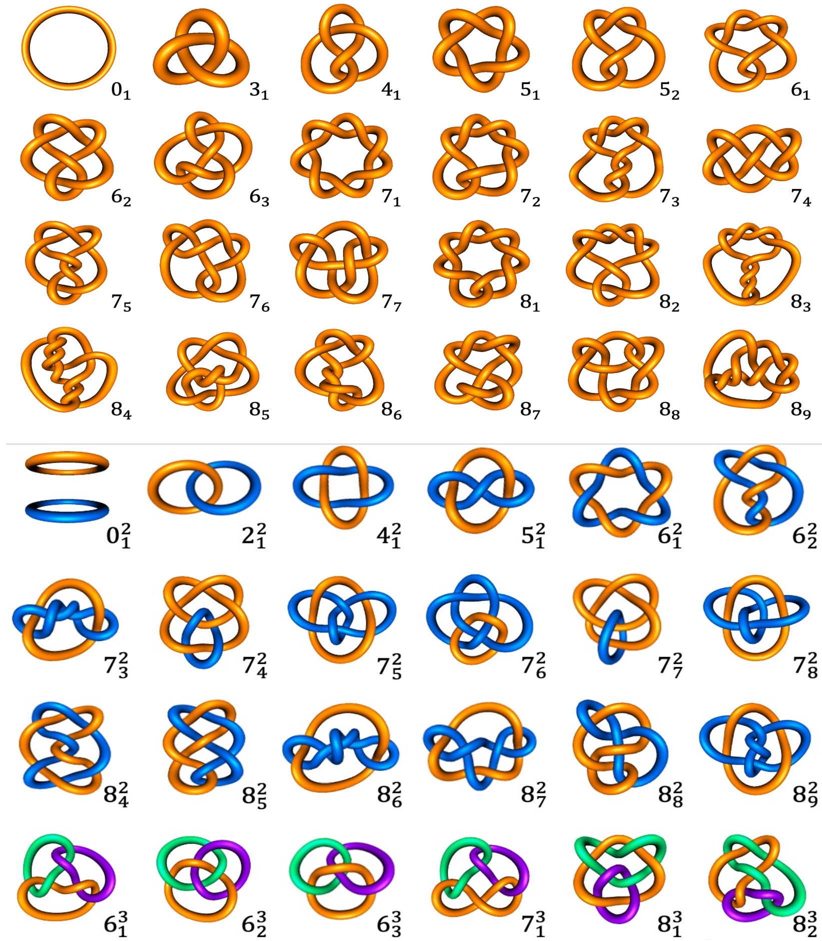


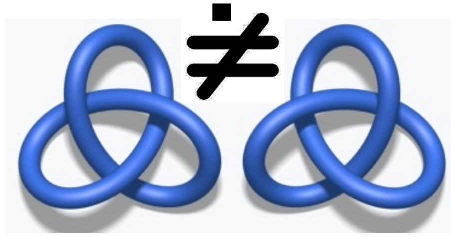
→Atoms as vortex knots in the aether

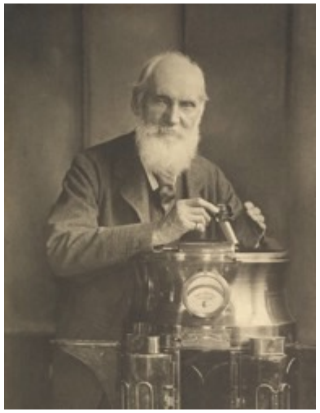
→Knots & multicomponent links

→Origins of the pure math's knot theory

→Concept of chirality – most knots are chiral!

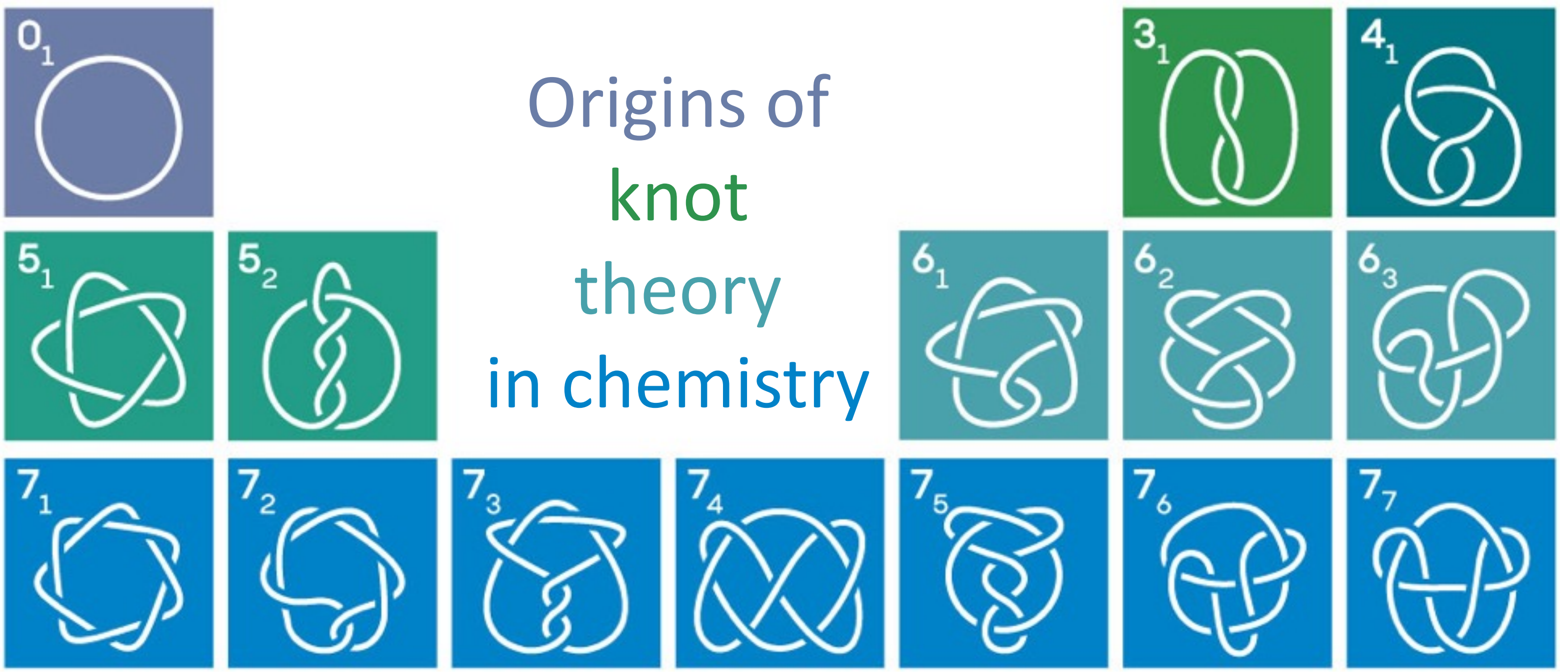
?

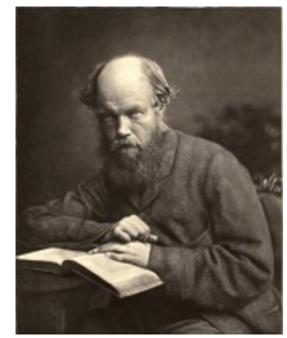




Lord Kelvin proposed ("On vortex knots," 1867) that the different chemical elements were manifestations of distinct knots tied in the luminiferous ether.

Origins of knot theory in chemistry

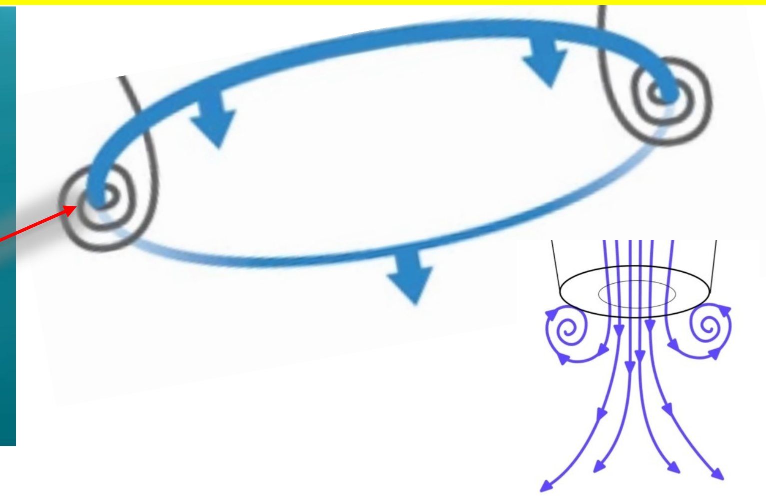




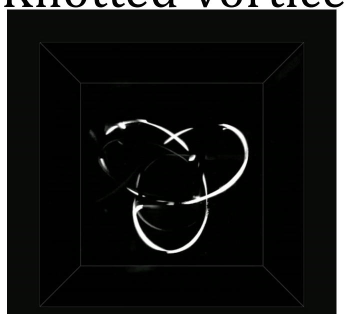
Prompted 1st enumeration of knots by Peter Tait.

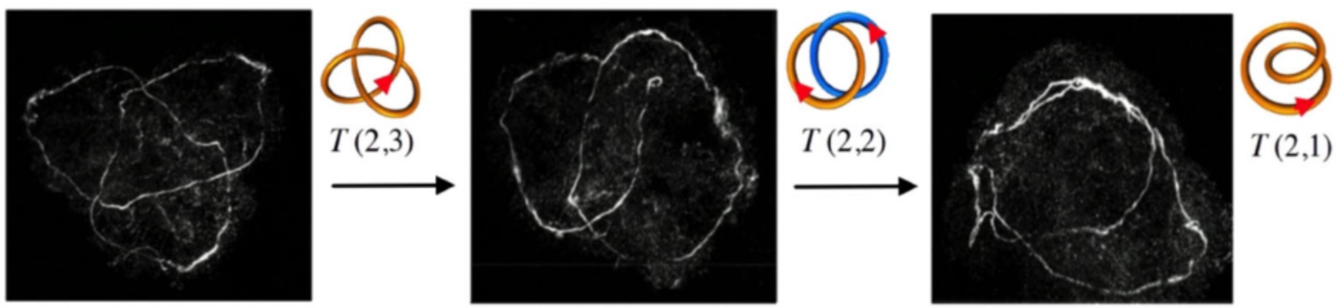
Quanta Magazine

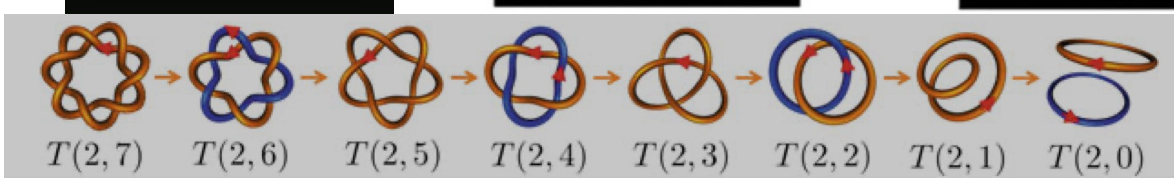
Vortex rings (unknots) & knots in water



→Knotted vortices decay in fluids through reconnection







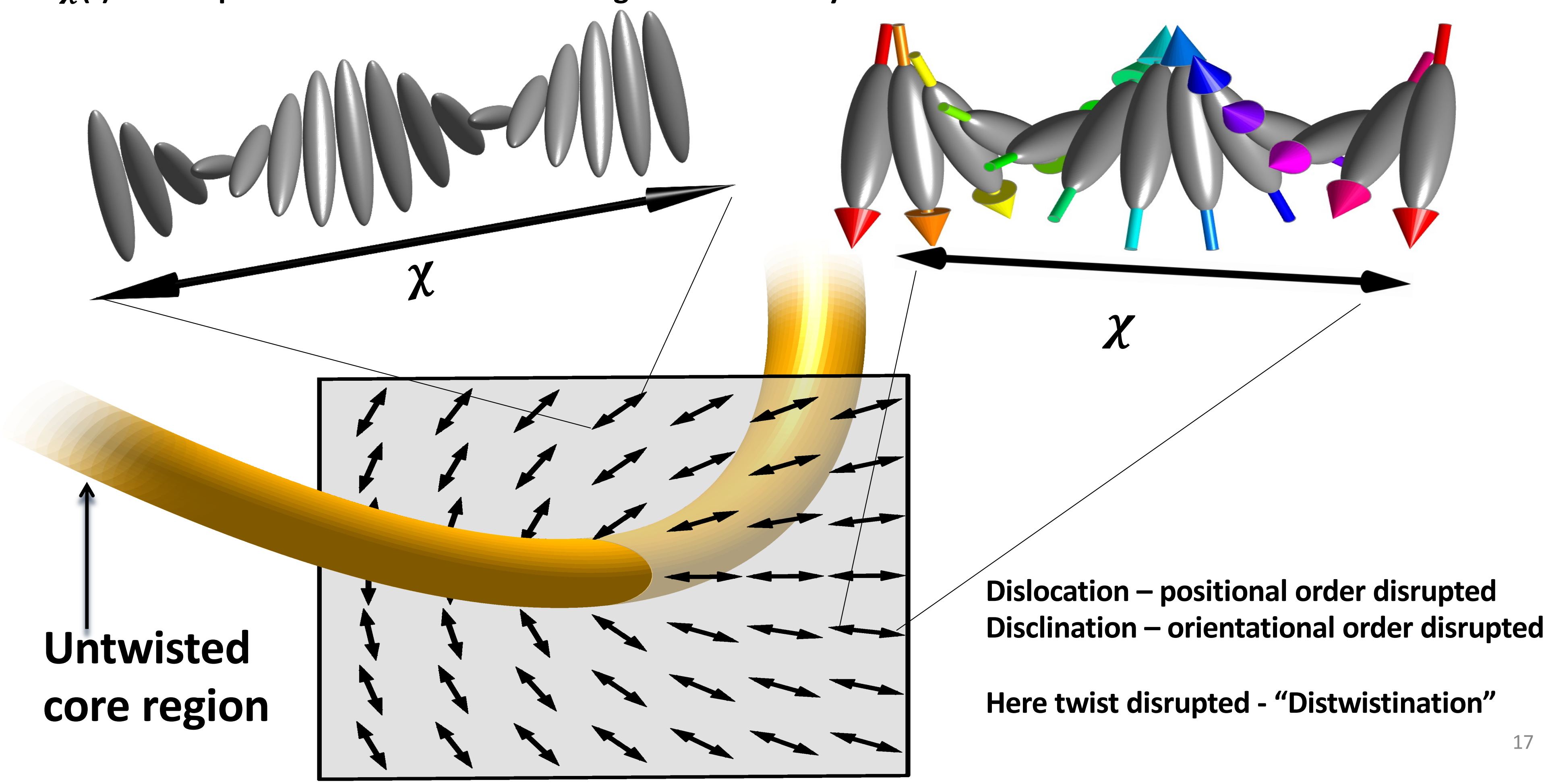
Kleckner & Irvine, *Nat. Phys.* **9**, 253–258 (2013)

Liu, X., Ricca, R.L. Sci. Rep. 6, 24118 (2016)
D Kleckner, LH Kauffman, WTM Irvine. Nature Physics 12, 650-655 (2016)
Kauffman, L. H. (eds Makaruk, H. E. & Owczarek, R. M.) 824, 47–66 (AMS, 2025).

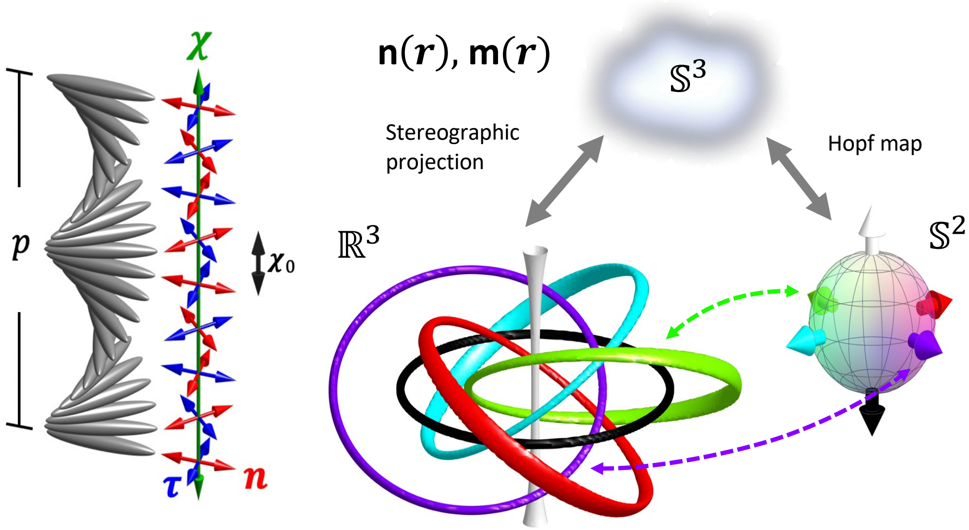
→Stable atom-like knots?

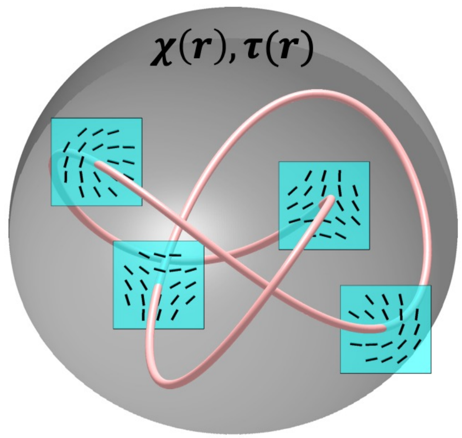
Vortex lines of disrupted twist in chiral (magnetic) LCs

χ(r) is a nonpolar for both nematic and magnetic colloidal systems



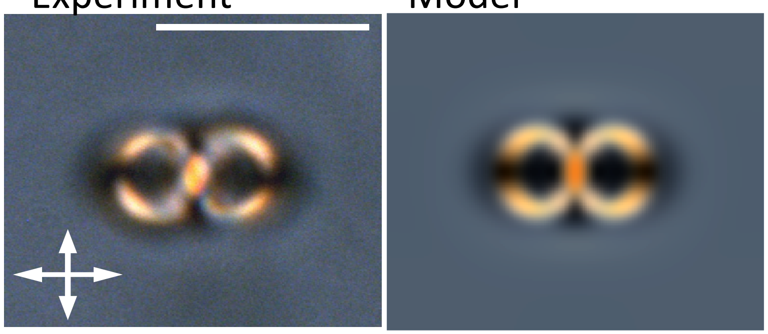
Hopfions in helical backgrounds: heliknotons





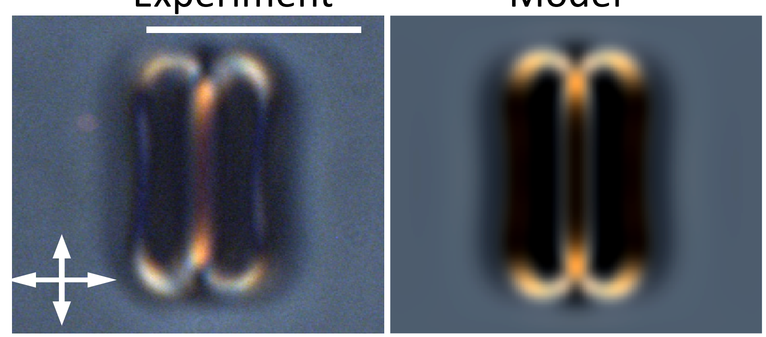
- **Hel**ical field **knot** soli**ton**
- Hopf soliton – linked closed-loop preimages!!!
- Knotted Singular vortex lines in $\boldsymbol{\chi}(\boldsymbol{r})$
- fractional skyrmions in $\mathbf{n}(\boldsymbol{r}), \mathbf{m}(\boldsymbol{r})$ forming a Hopf soliton

Experiment Model



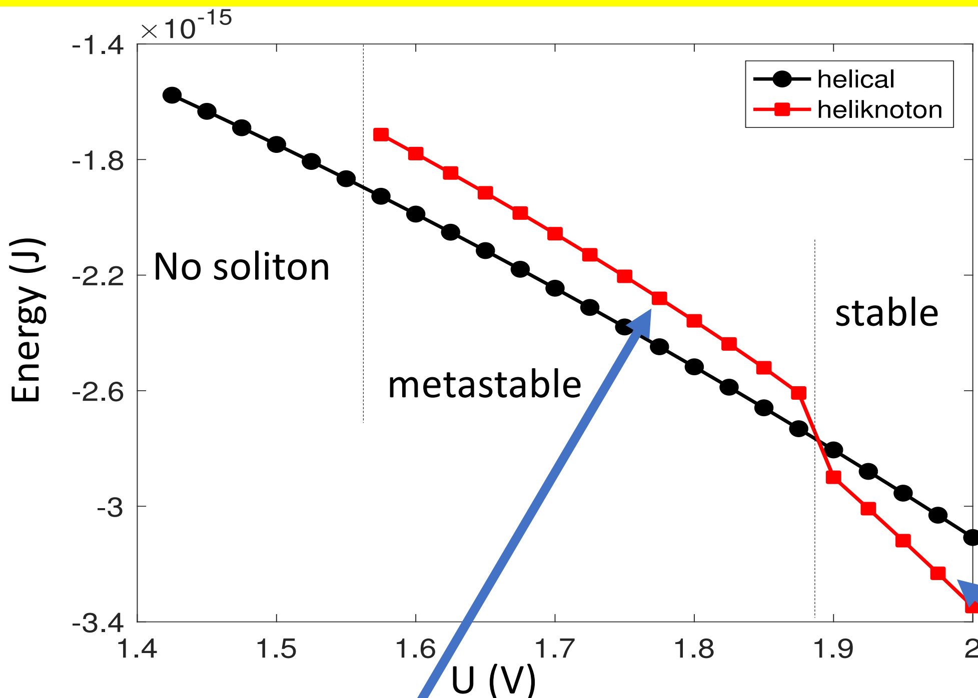
Increase applied voltage by ~1V

Experiment Model



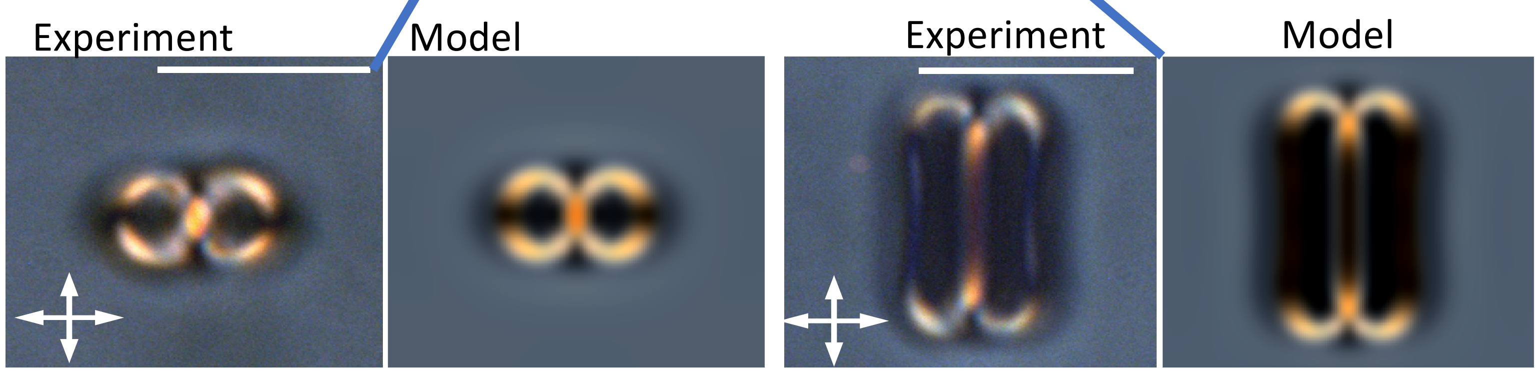
JSB Tai, II Smalyukh, *Science* **365**, 1449 (2019).

Individual heliknoton: stability & eelectric morphing



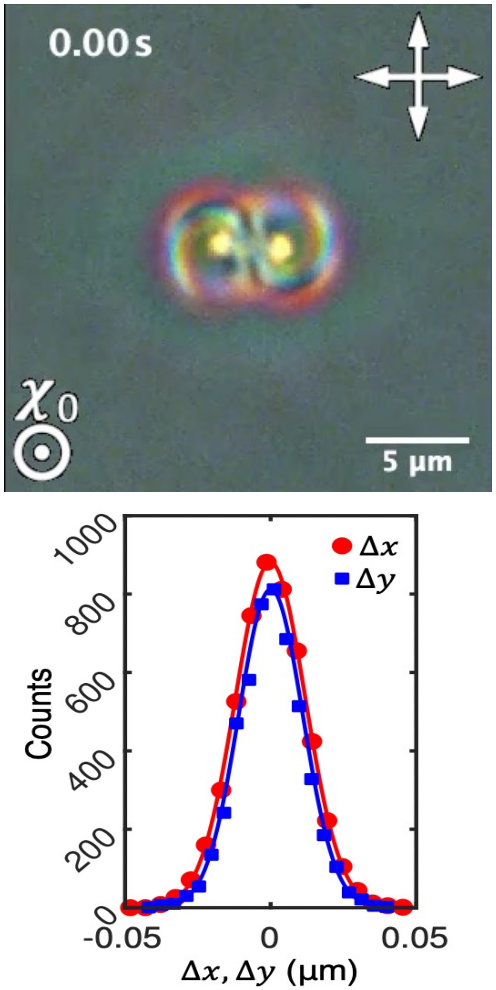
→Electric field coupling free energy term:

$$f_e = -\frac{\varepsilon_0 \Delta\varepsilon}{2} \int (\mathbf{E} \cdot \boldsymbol{n})^2$$

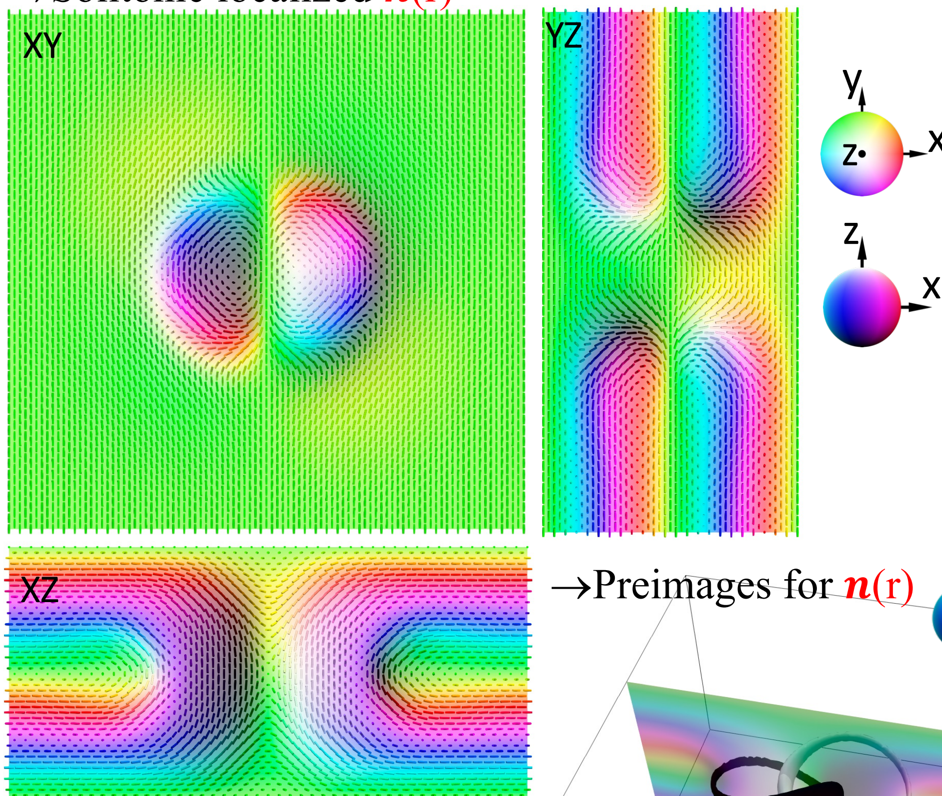


Knot solitons in a helical background: heliknotons

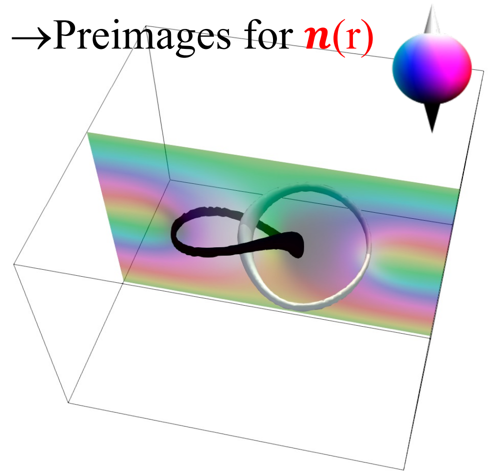
→Helical far-field:



→Solitonic localized $\boldsymbol{n}$(r)

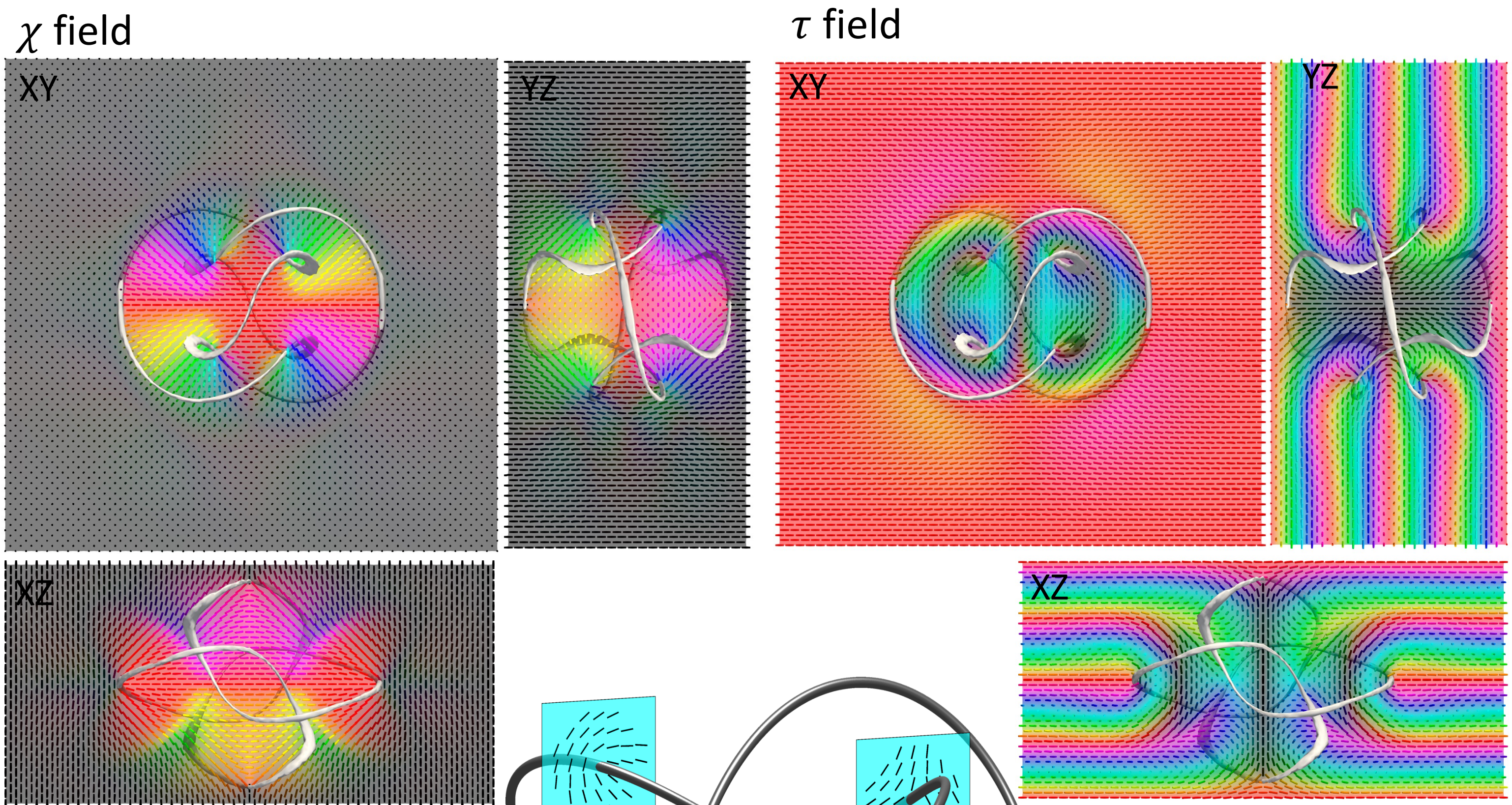


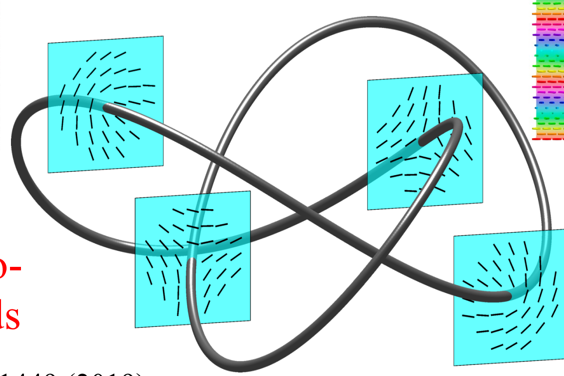
→Preimages for $\boldsymbol{n}$(r)



→Q=1 Hopf soliton in $\boldsymbol{n}$(**r**)

Torus knot vortices in the immaterial χ & τ **fields**

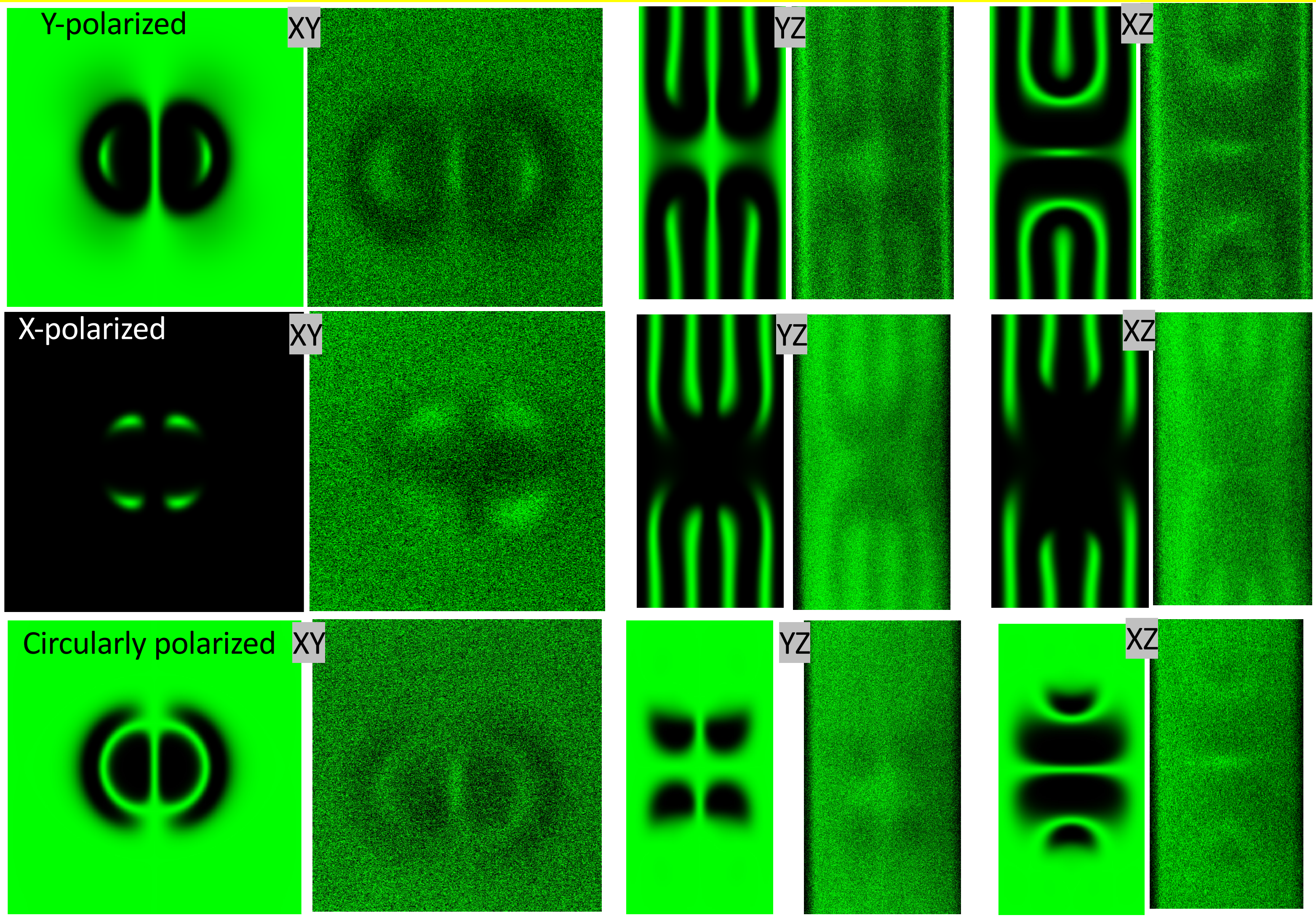




→Trefoil knots of singular co-located vortices in χ & τ fields

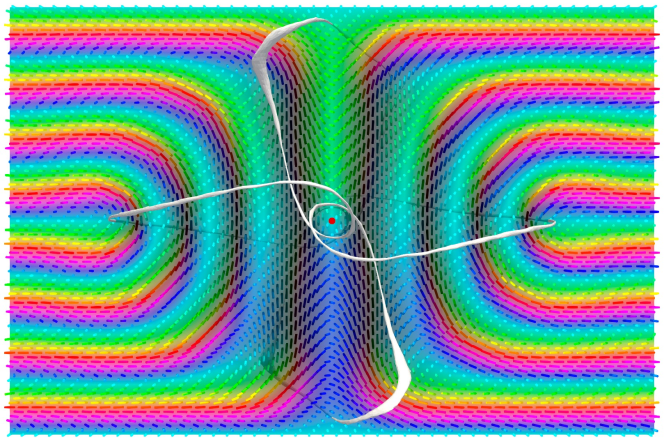
JSB Tai, II Smalyukh, *Science* **365**, 1449 (2019).

Mapping the fields in 3D & comparing to the model

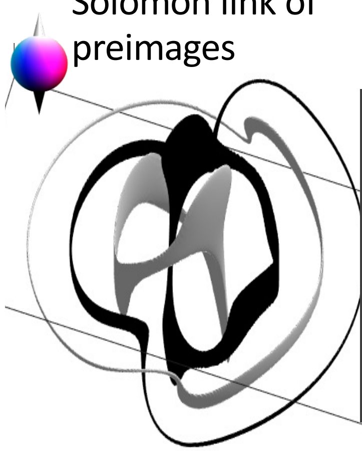


High-degree heliknotons

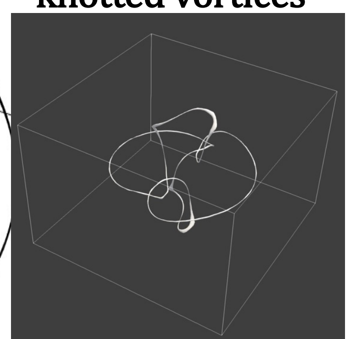
Linked preimages with Q=2 in $\boldsymbol{n}(\mathrm{r})$



Solomon link of preimages



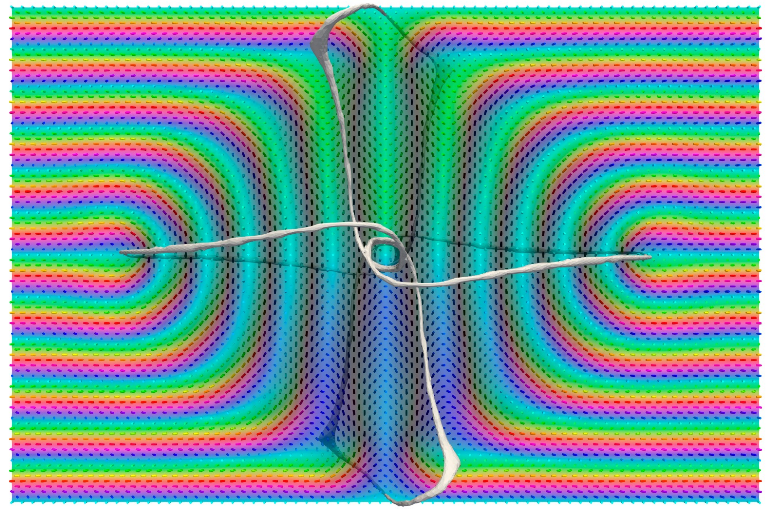
χ & τ **fields with knotted vortices**



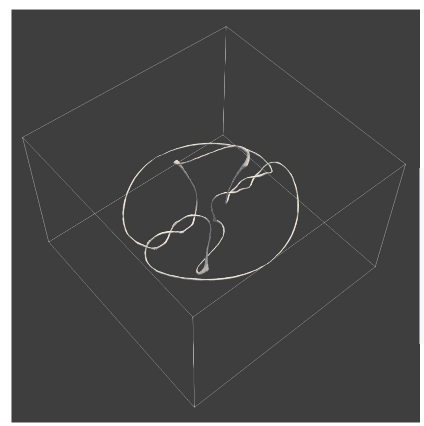
Cinquefoil 51 torus knot

N_v=2Q+1

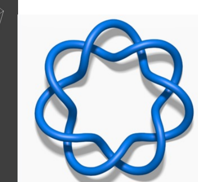
Linked preimages with Q=3 in $\boldsymbol{n}(\mathrm{r})$



χ & τ **fields with knotted vortices**

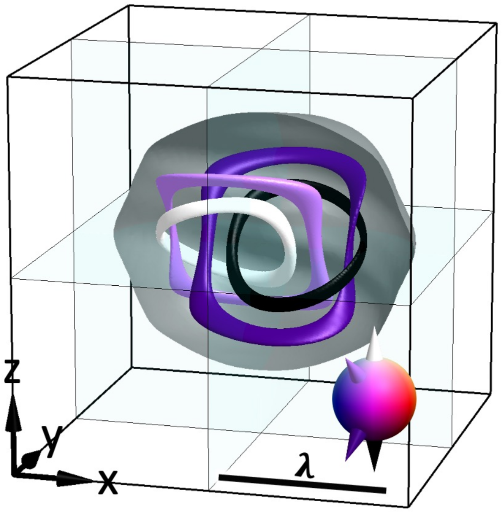


7_1 torus knot

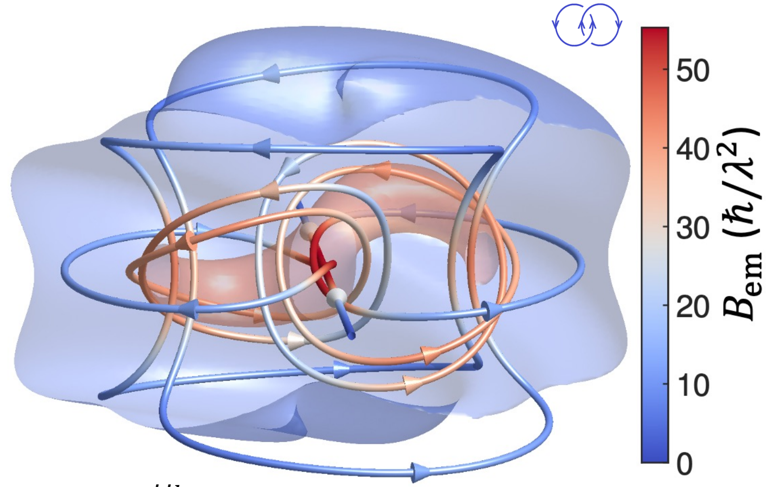


Solid-state magnetic heliknoton & knotted emergent field

→Linked preimages in magnetization field **m**(**r**)

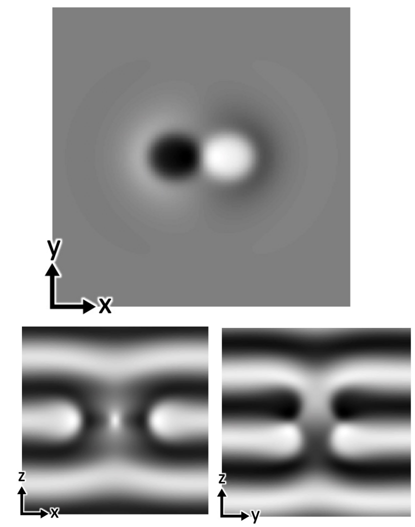


→Knotted emergent magnetic field

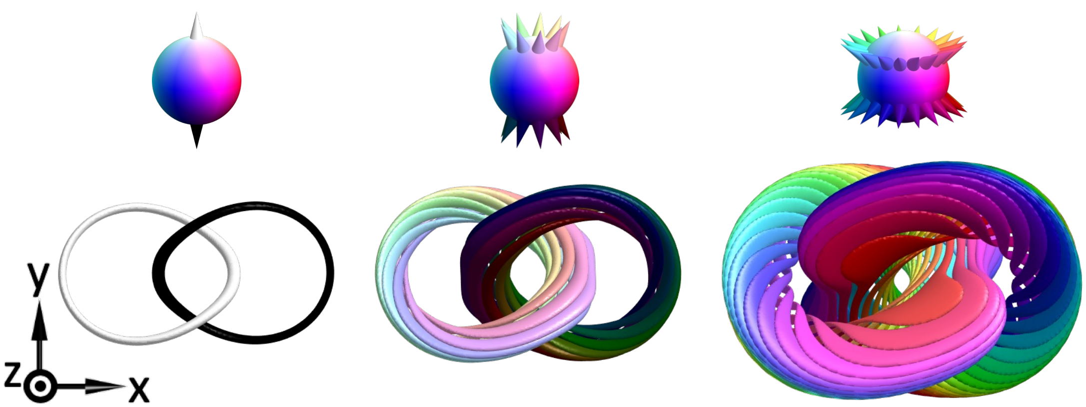


$$(B_{em})_i = \hbar\varepsilon^{ijk}\boldsymbol{m}\cdot(\partial_j\boldsymbol{m}\times\partial_k\boldsymbol{m})/2$$

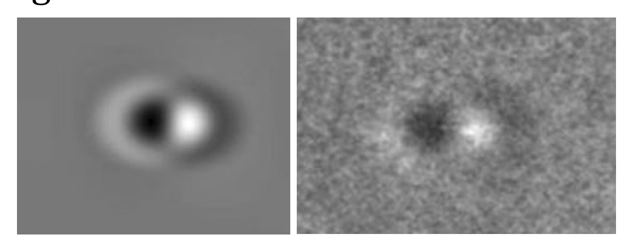
Lorentz TEM images



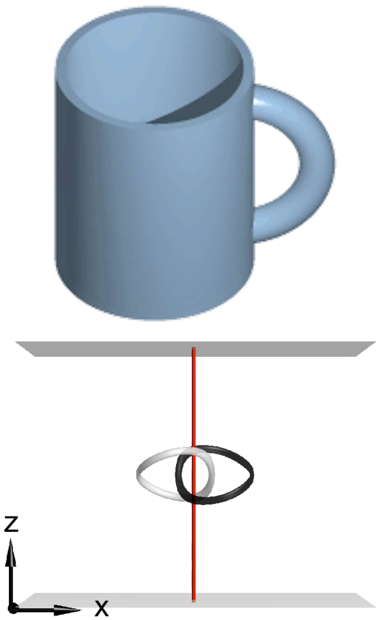
R. Voinescu, J.-S. B. Tai and I. I. Smalyukh. *Phys Rev Lett* **125**, 057201 (2020)

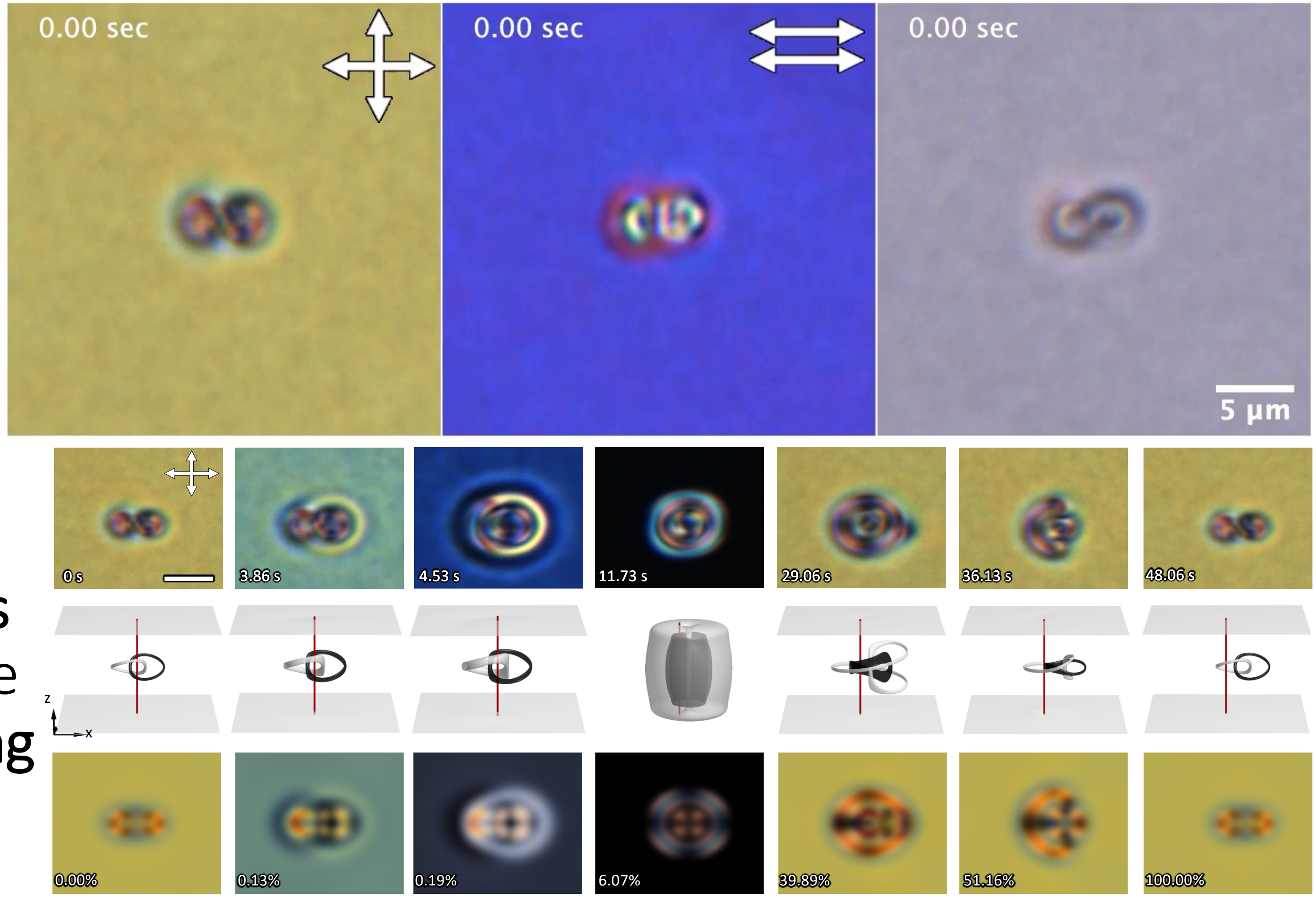


→Long Li et al. Nature Materials (2026)
doi.org/10.1038/s41563-025-02450-0



Topology-preserving switching at higher voltages (~4V)

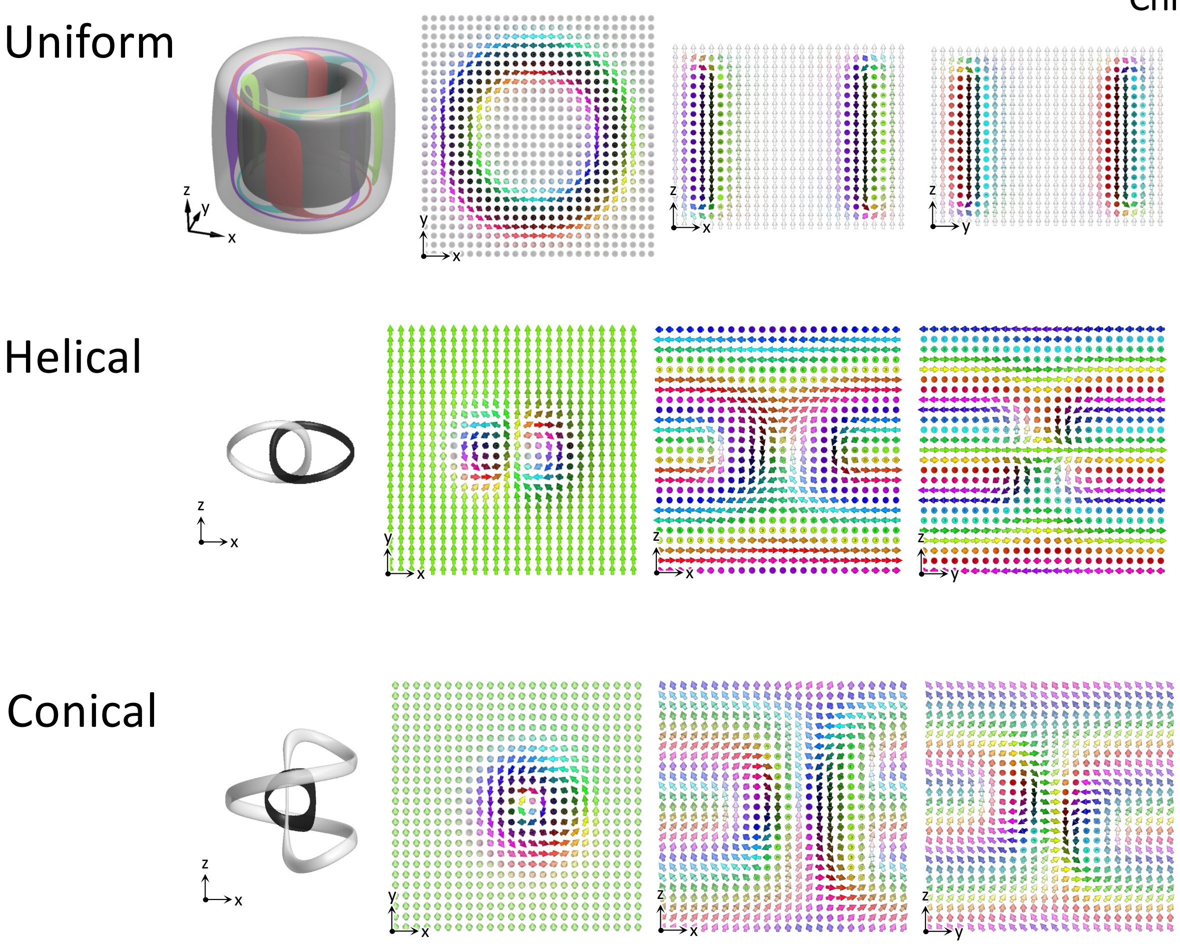


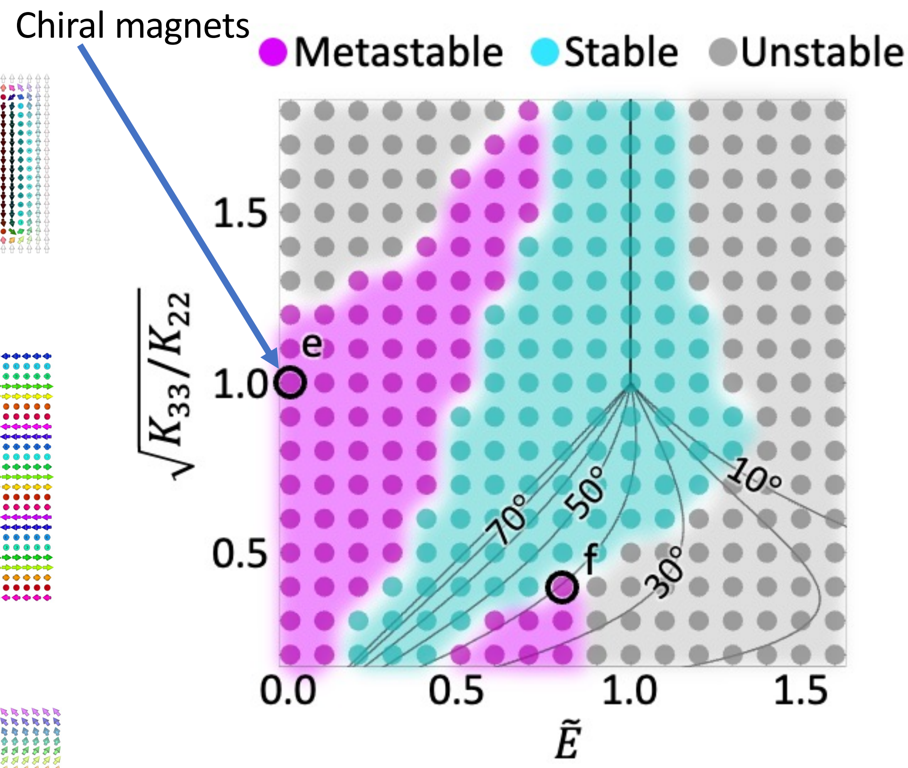


experiments closely agree with **modeling**

J Tai, J Wu & I Smalyukh. *NATURE COMMUNICATIONS* **13**, 2986

Stable in an electrically switchable background



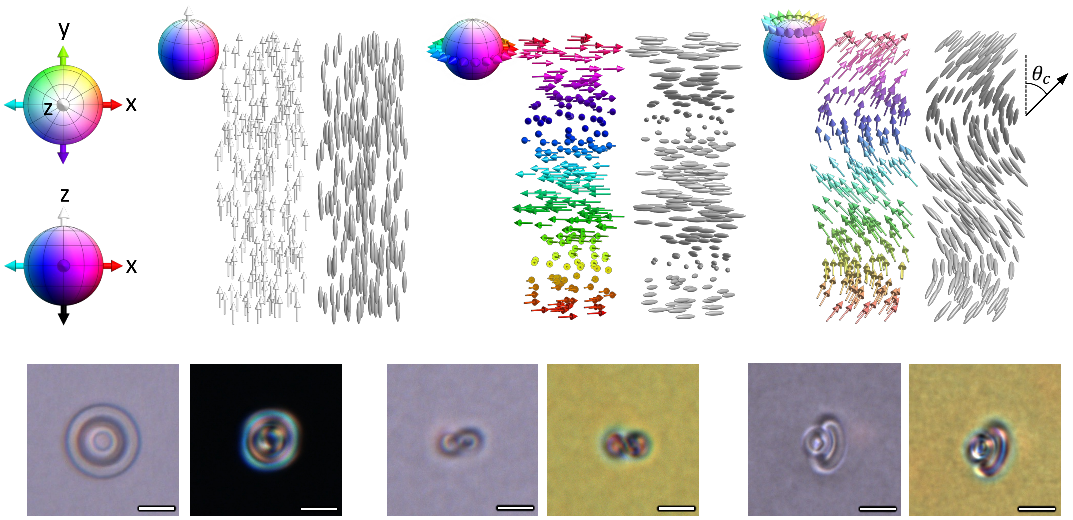


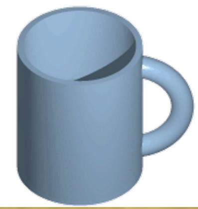
J Tai, J Wu & I Smalyukh. *NATURE COMMUNICATIONS* **13**, 2986

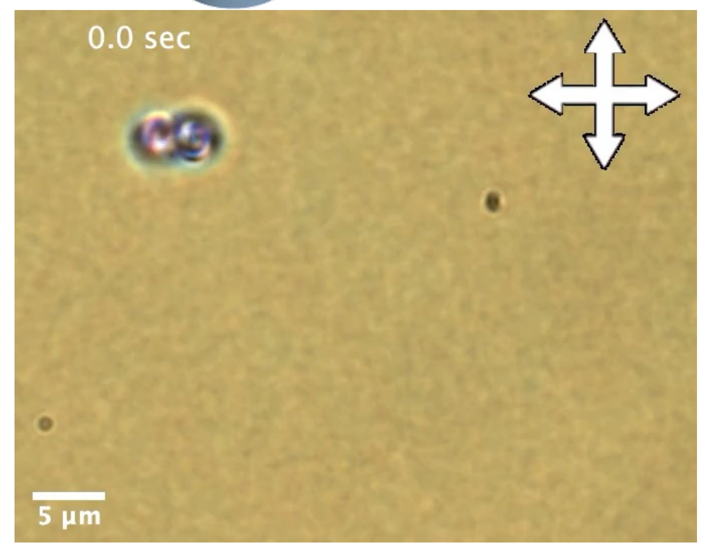
Nonreciprocal evolution of fields & hopping of hopfion

→Analogous to morphing surfaces without changing genus

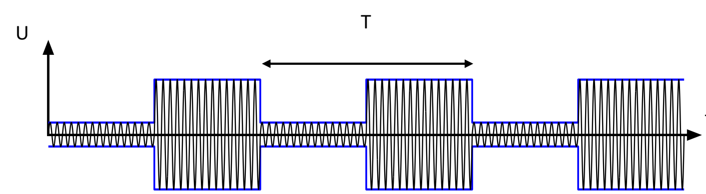
→Topology-preserving hopfion-heliknoton morphing while smoothly embedded in the uniform/conical-helical background:







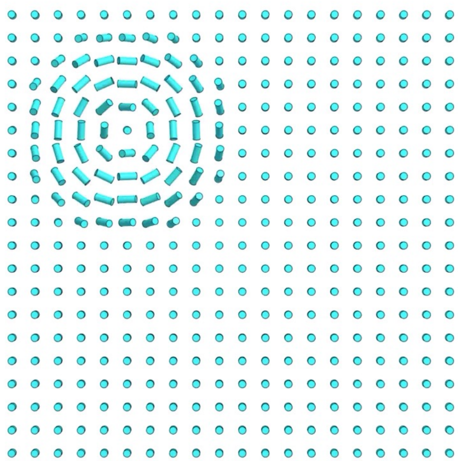
→Nonreciprocal field evolution yields hopping-like 3D trajectory of motion in response to a periodically oscillating field



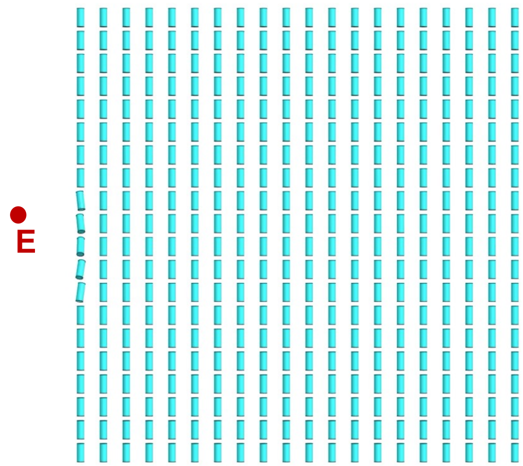
J Tai, J Wu & I Smalyukh. *NATURE COMMUNICATIONS* **13**, 2986

Liquid crystals with “activated” solitons, like waves

→ At low modulated U

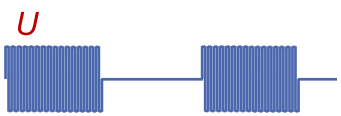


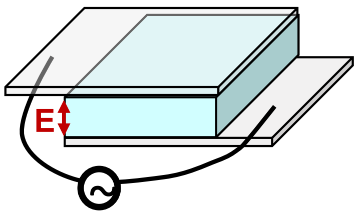
→ At higher modulated U, after coarsening



→ Analogous to a moving stadium wave

→Solitons have no mass, overdamped motion



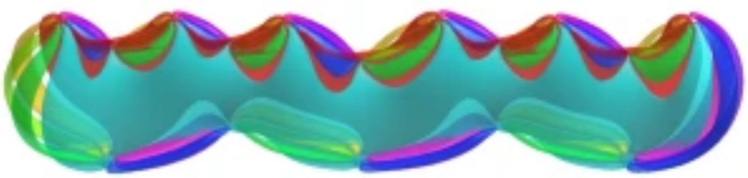


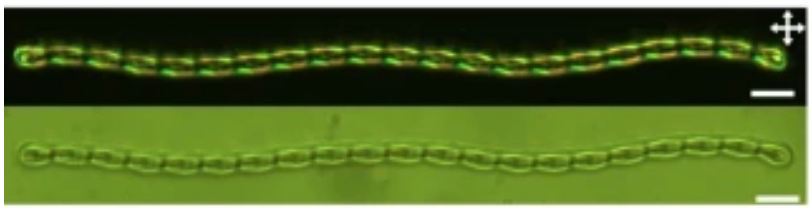
→Local conversion of energy

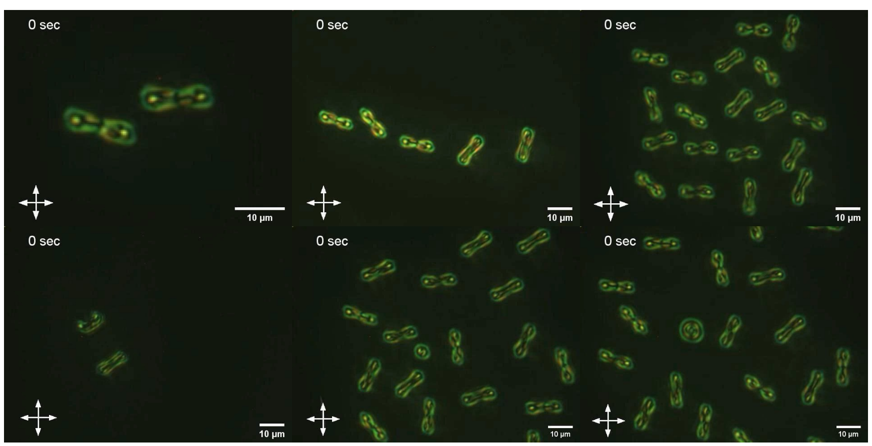
→Rotational dynamics alone can lead to soliton translation

H. Zhao, J.-S. B. Tai, J.-S. Wu and I. I. Smalyukh. Nature Physics **19**, 451-459 (2023)

Topological molecules/polymers & their dynamics



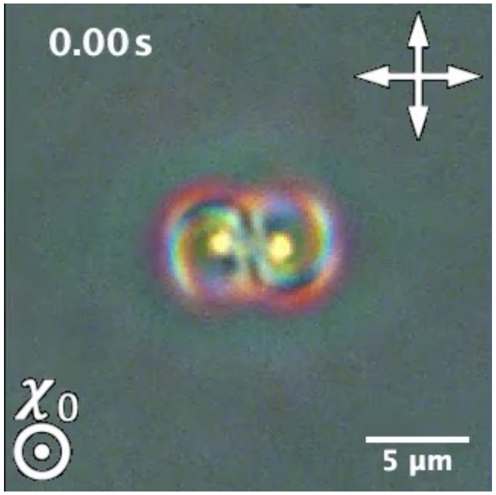


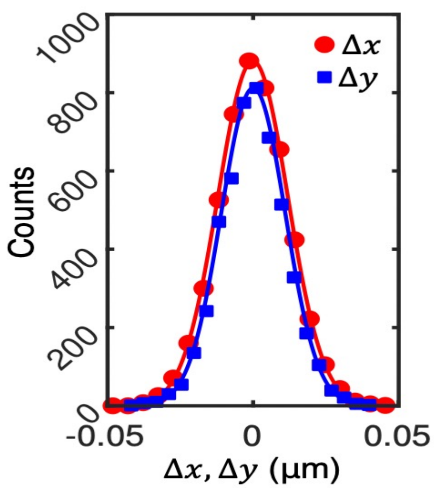


Zhao, Malomed & Smalyukh. N*ature Comm* **14**, 4581 (2023)

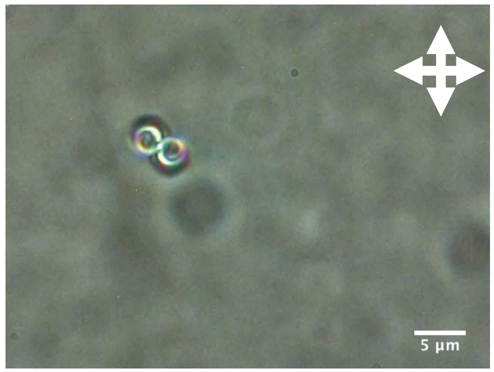
3D localization/motion & self-assembly

→Helical far-field:

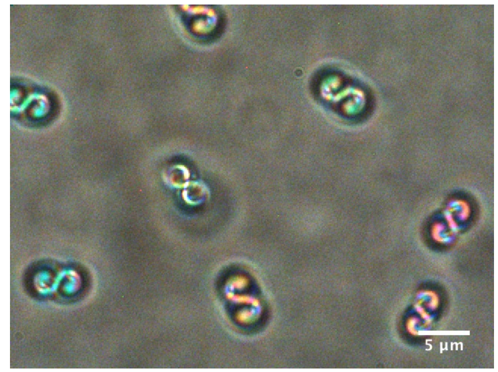


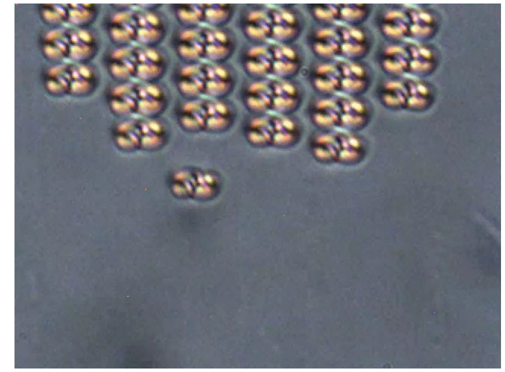


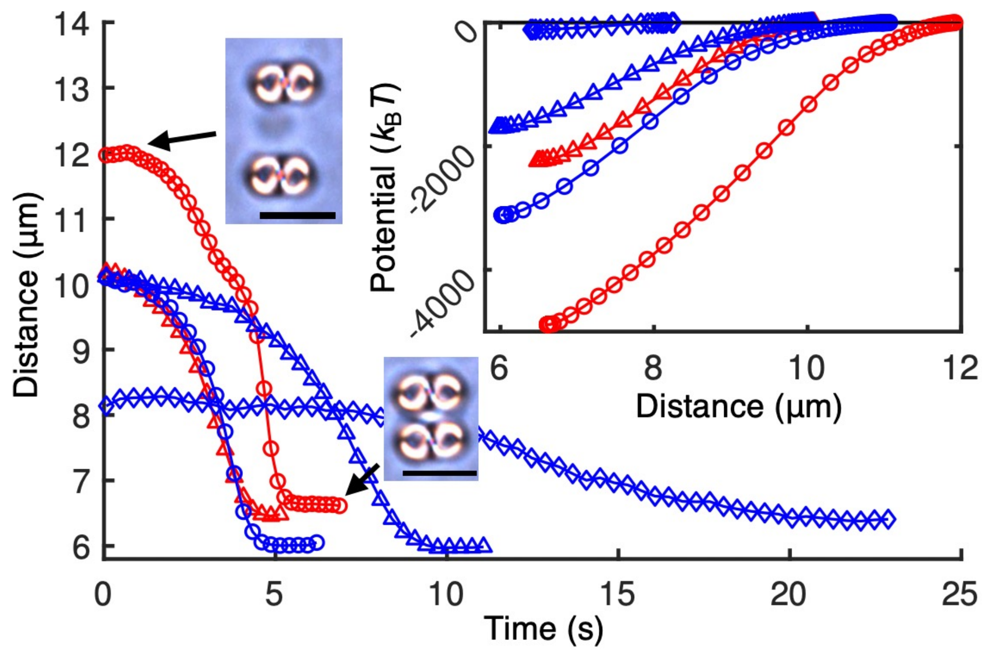
Rotations & 3D translations



Gas of heliknotons



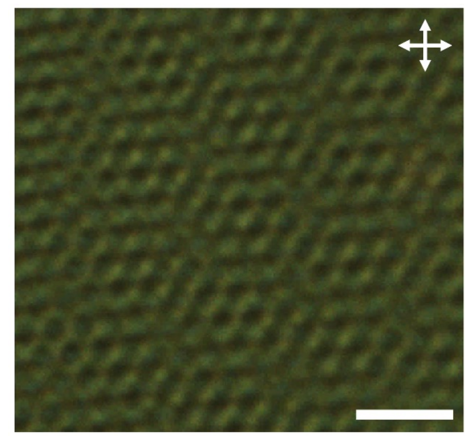




→Anisotropic 3D interactions tuned within (10-10,000)k_BT

→ Soliton gas and crystals

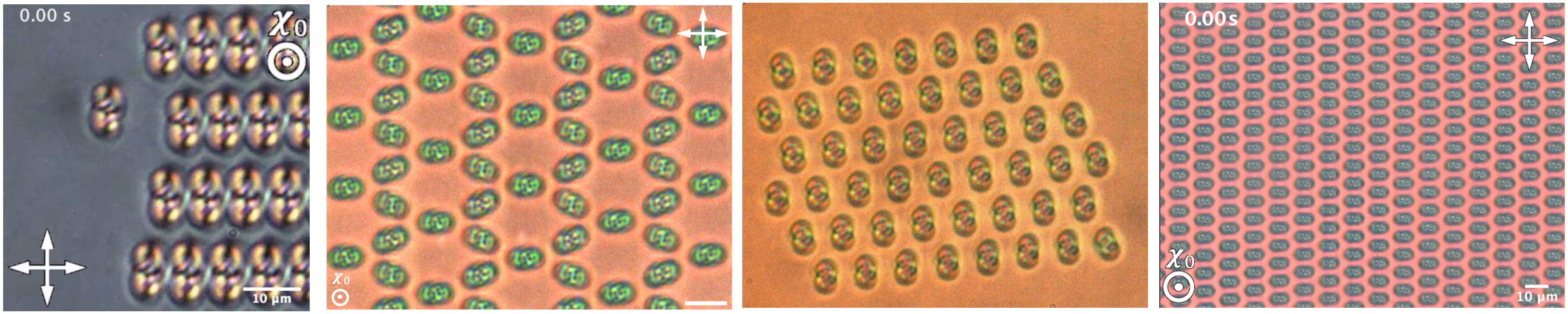
→Control metaatom interaction



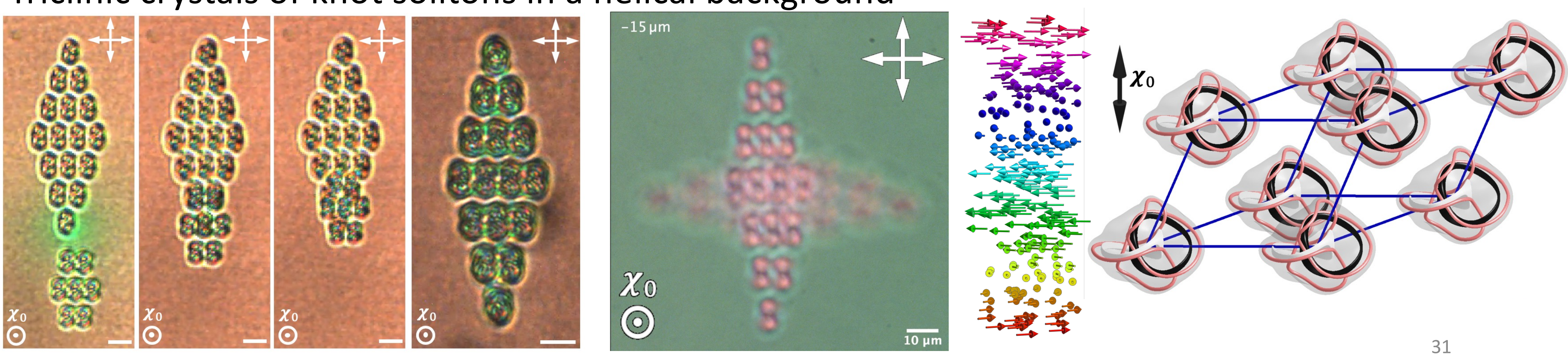
H Zhao & I Smalyukh, (2026).

Crystals of heliknotons & electrostriction

Self-assembly of close-packed & open crystals, GIANT electrostriction



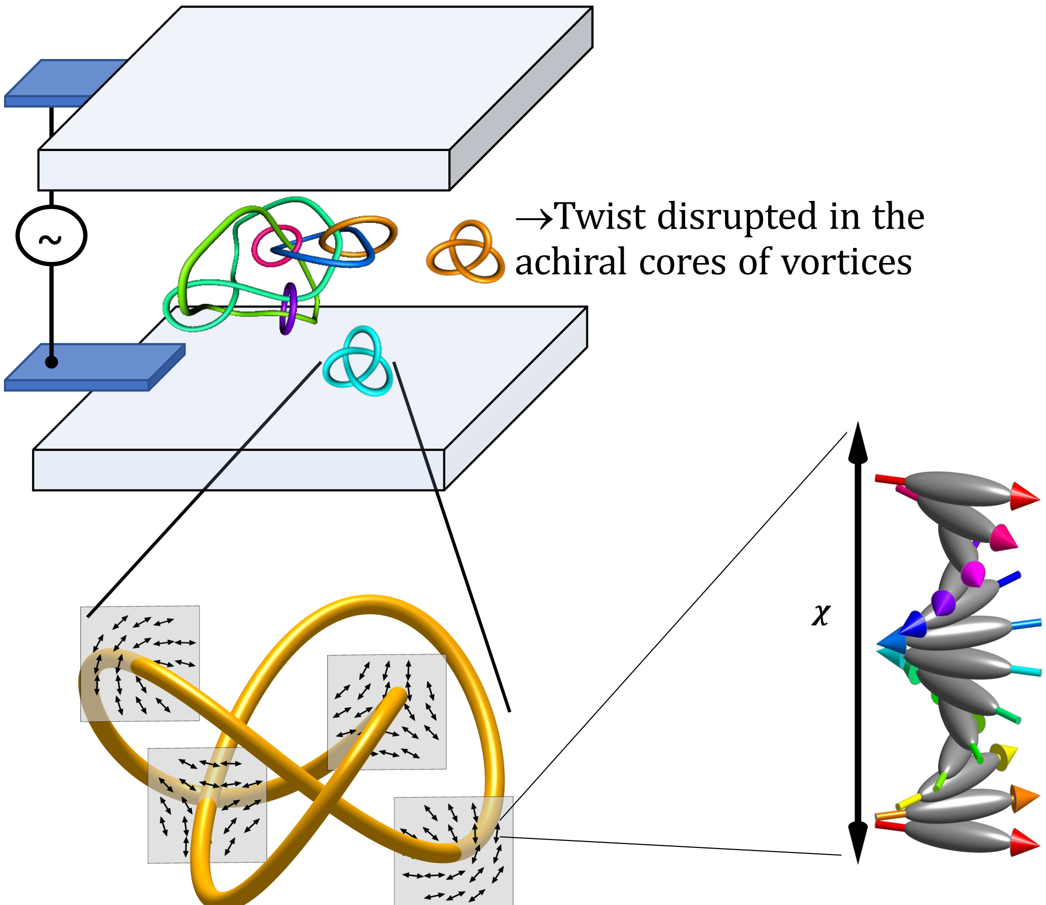
Triclinic crystals of knot solitons in a helical background



J Tai & I Smalyukh, *Science* **365**, 1449 (2019).

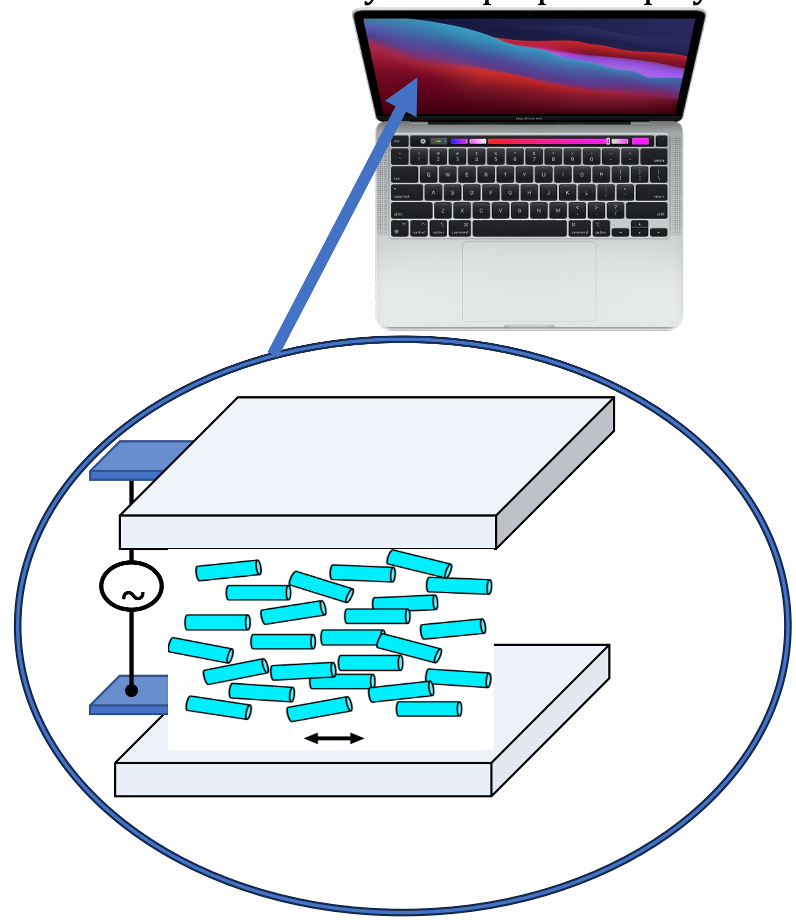
Electric control of vortex knots in a chiral LC

→Chiral LC with vortex knots between transparent electrodes



→Twist disrupted in the achiral cores of vortices

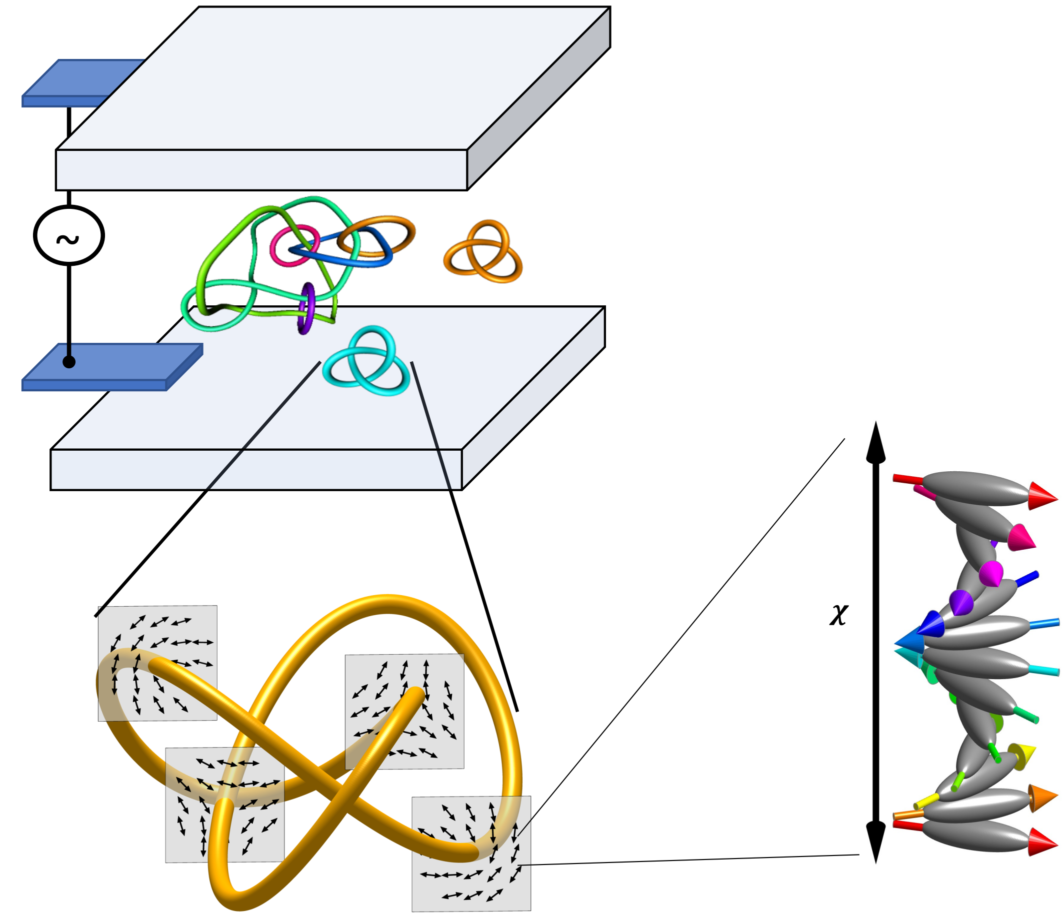
→Like in your laptop's display



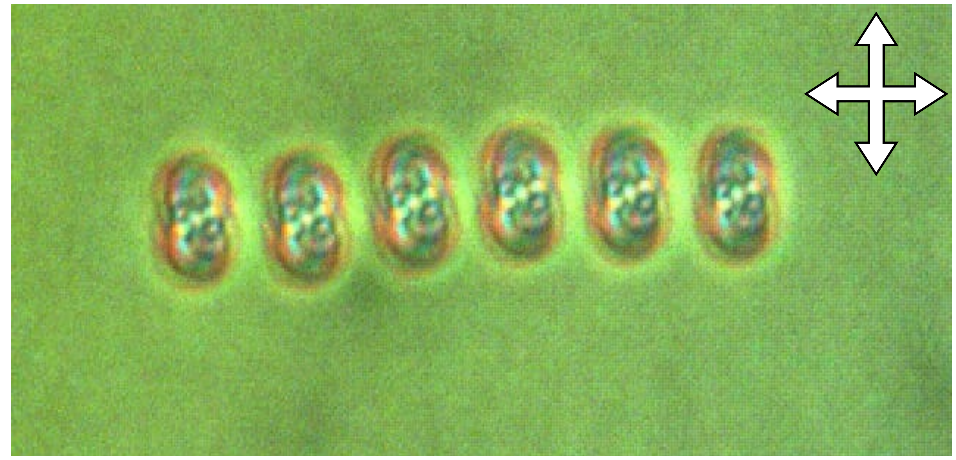
→Chiral nematic & colloidal magnet LC media as artificial "eather"?

Electric control of vortex knots in a chiral LC

→Apply AC field



→Apply 5 V, at 1 kHz AC field ● **E**

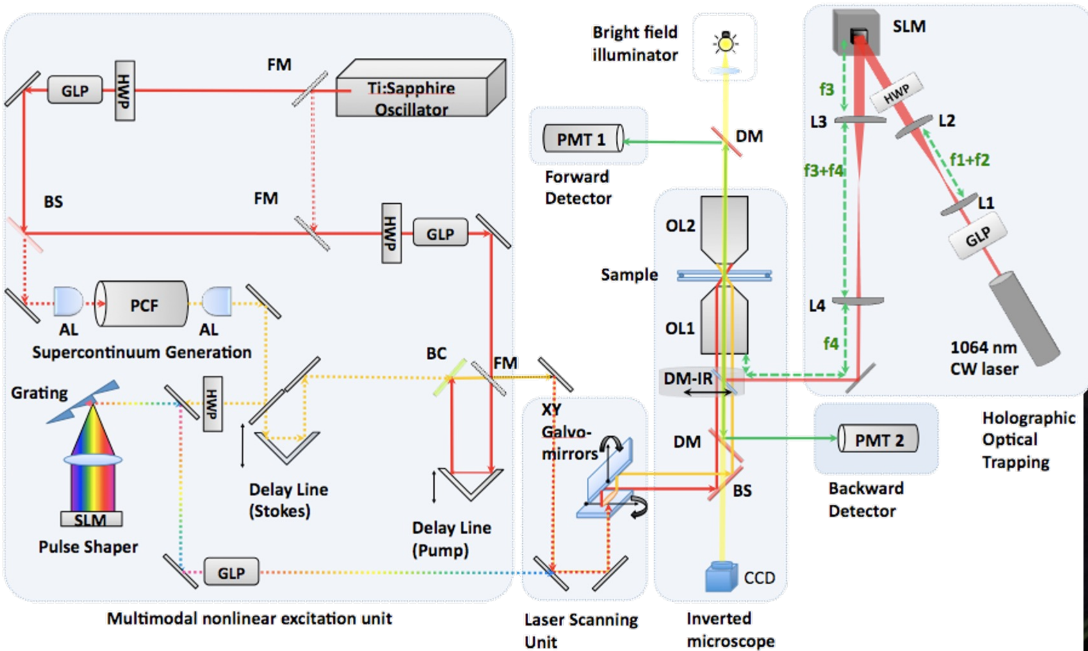


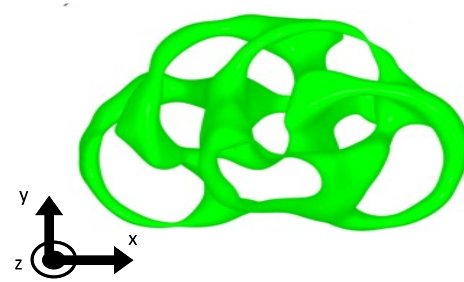
→Lines of disrupted twist/chirality make chiral knots

→What happens to knots in the “stronger” applied field?

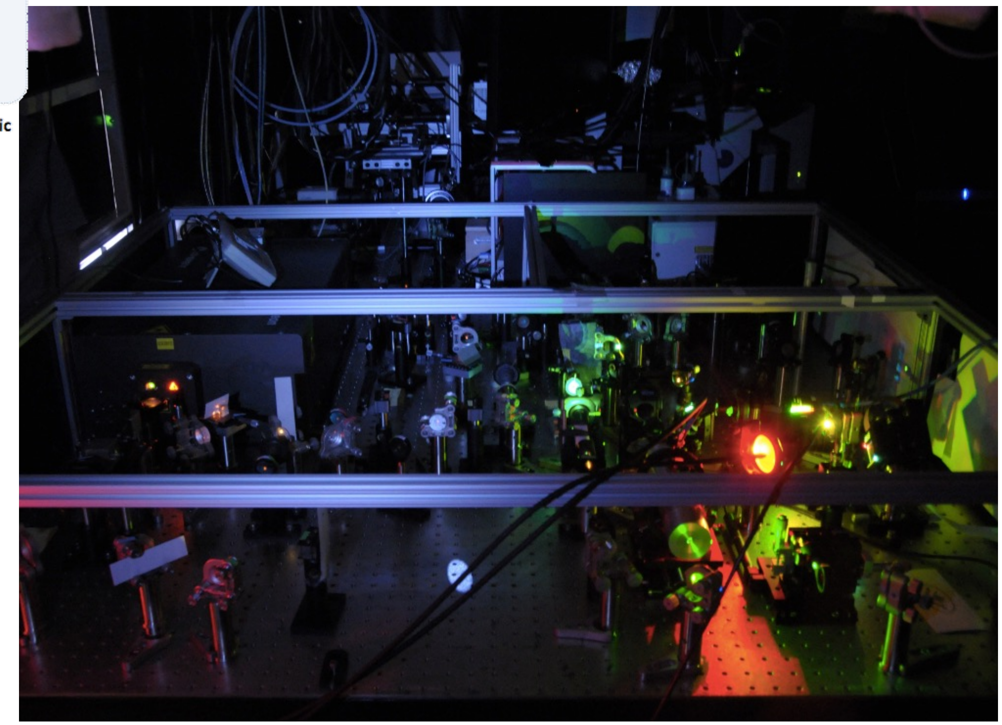
D. Hall, J Tai L Kauffman & I Smalyukh. *Nature Physics* **22**, 103–111 (2026)

3D nonlinear optical imaging setup & reconstruction



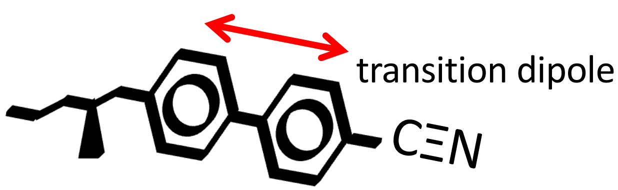


→Polarization-dependent 3D luminescence images



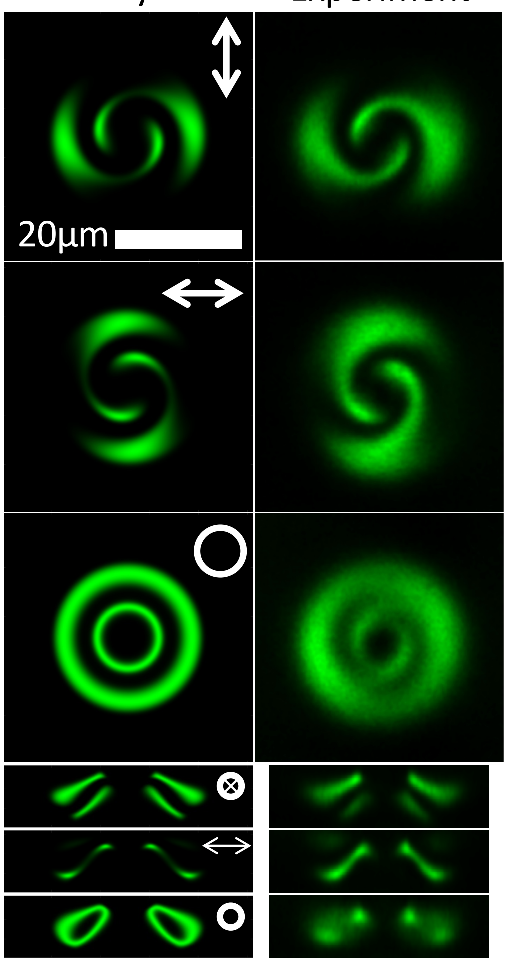
→**Orientation-sensitive 3D nonlinear optical imaging**

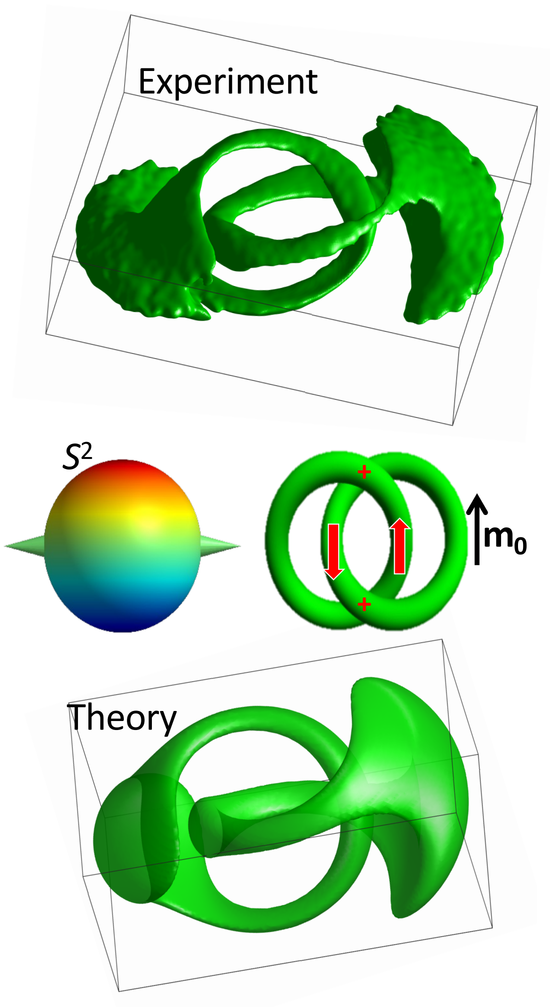
→**Map fields in 3D with sub-diffraction limited resolution**



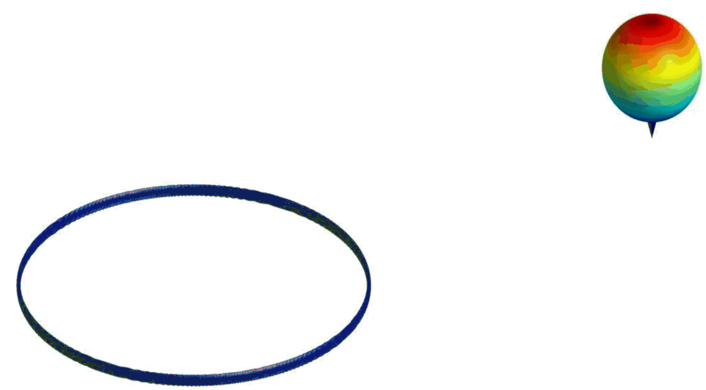
T. Lee, R. P. Trivedi & I. I. Smalyukh, *Opt. Lett.* 35, 3447

Numerical modeling versus experiments





→**Filling localized space with all preimages:**

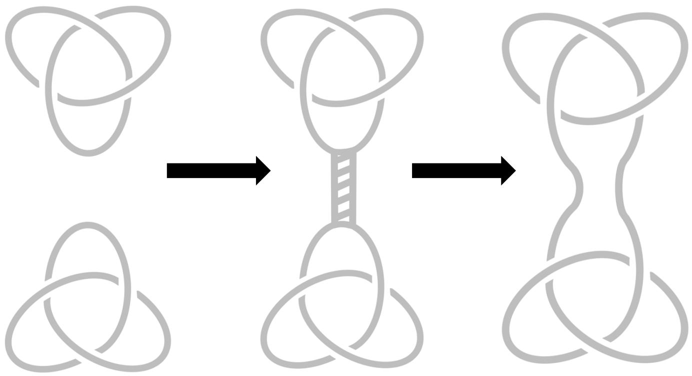


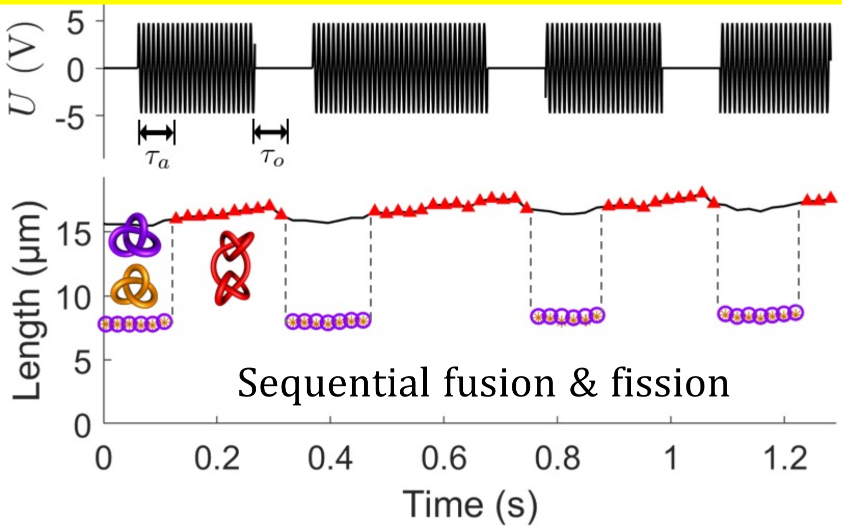
P.J. Ackerman & I.I. Smalyukh. *Nature Mater.* **16**, 426-432 (2017)

D. Hall, J Tai L Kauffman & I Smalyukh. *Nature Physics* **22**, 103–111 (2026)

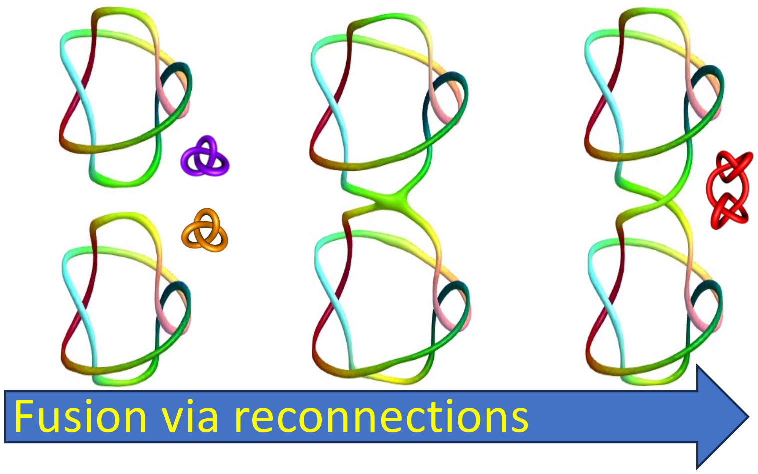
Switching the "connected sum" of vortex knots

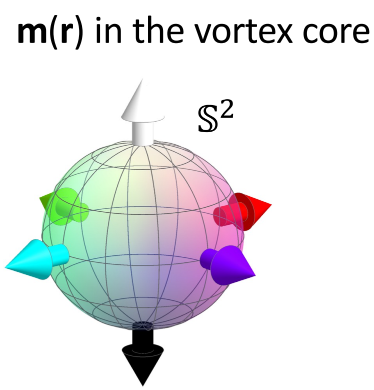
→Pure math, knot theory: connected sum of knots

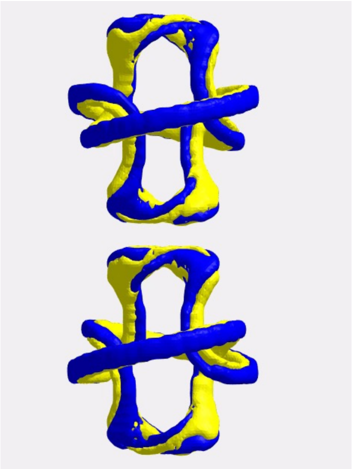


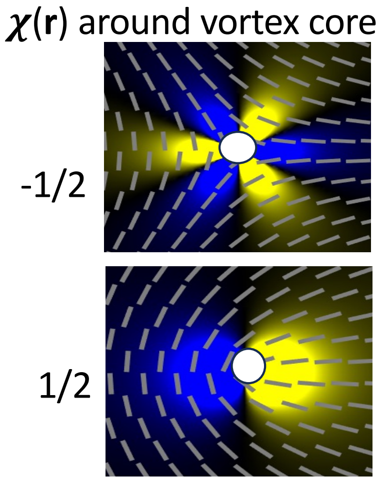


→Number of under/over crossings is added

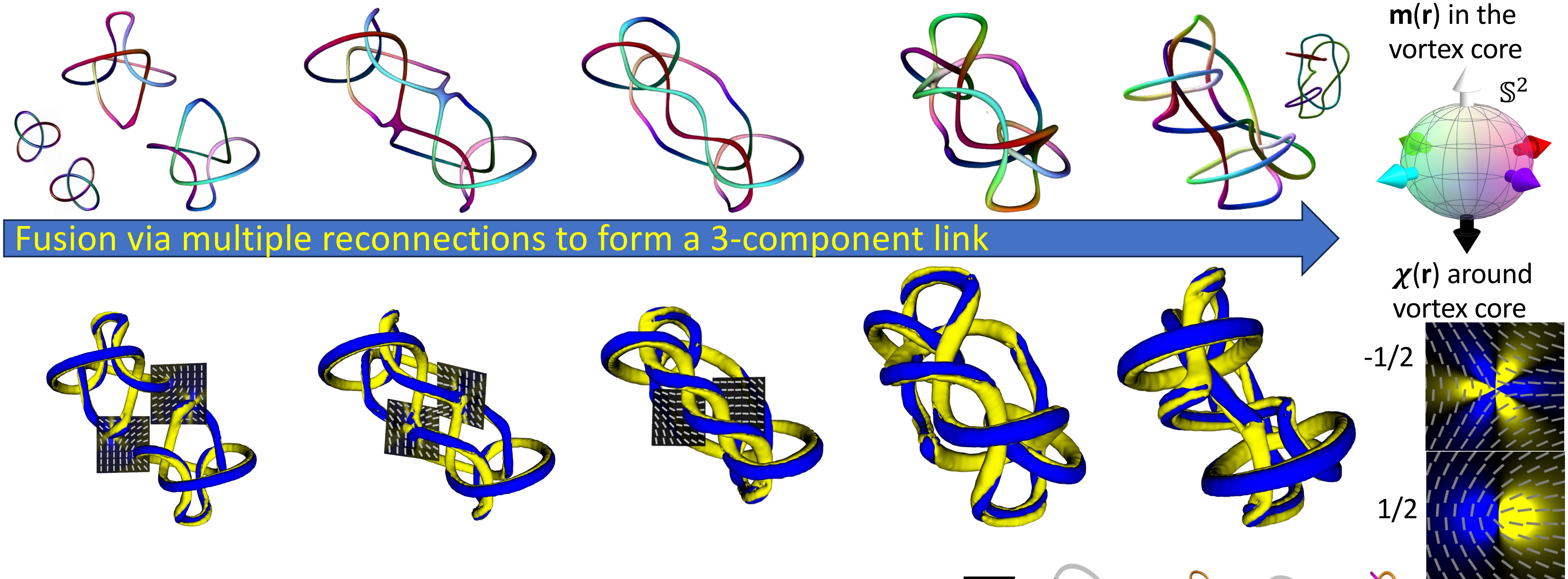






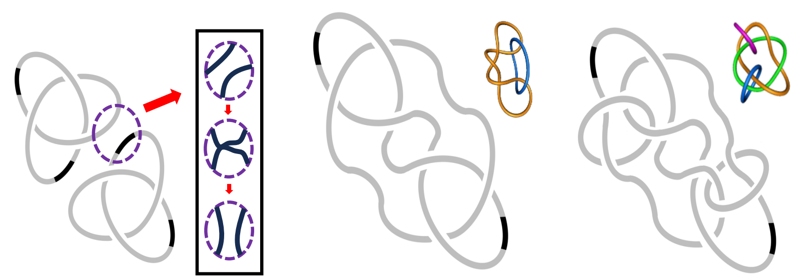


Vortex knot “double” reconnections



→2 trefoils re-connect into a 2-component link

→…and then into a 3-component link

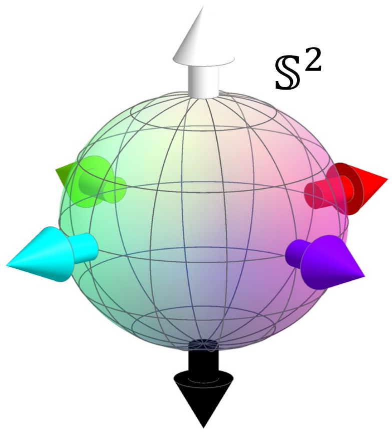


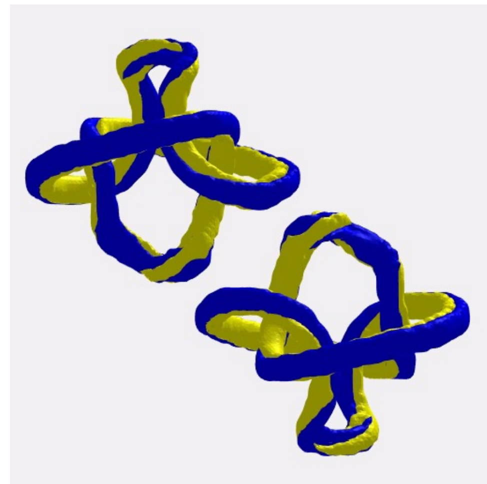
D. Hall, J Tai L Kauffman & I Smalyukh. *Nature Physics* **22**, 103–111 (2026)

Visualizing vortex analogs of nuclear fusion & fission

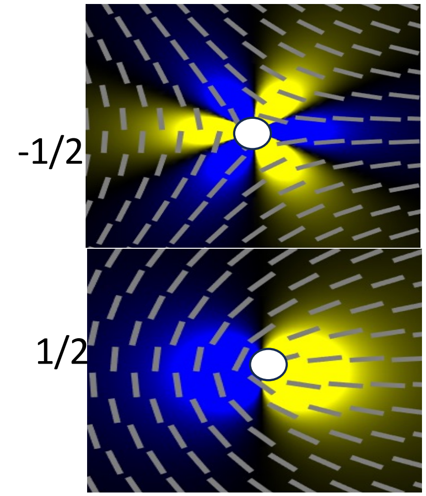
→**Subsequent fusion & fission processes induced by a pulse of electric field**

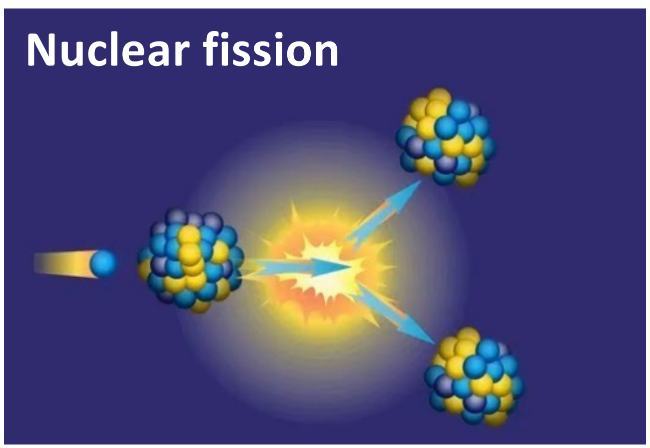
m(**r**) in the vortex core

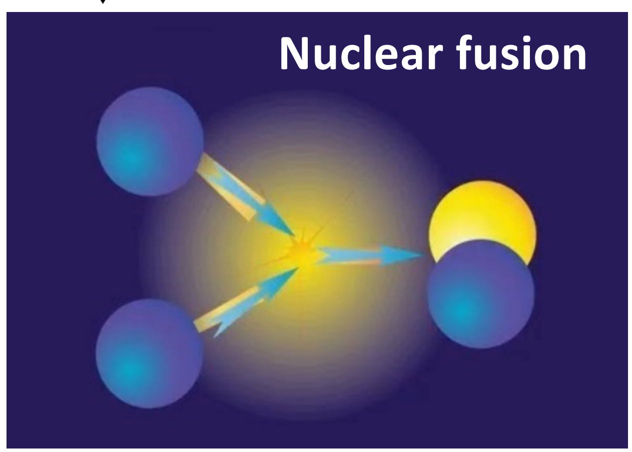


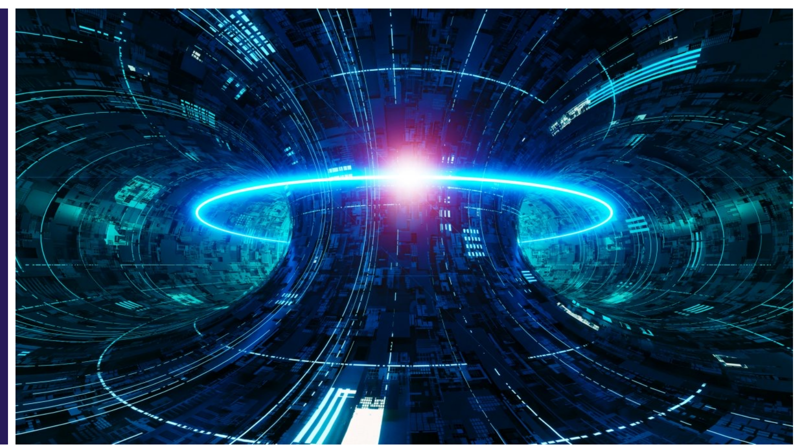


χ(**r**) around vortex core



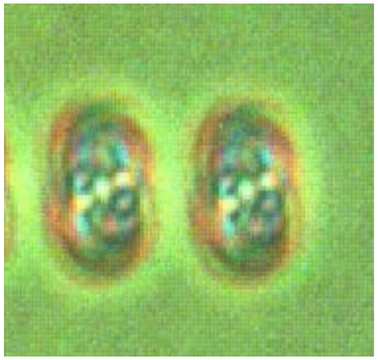


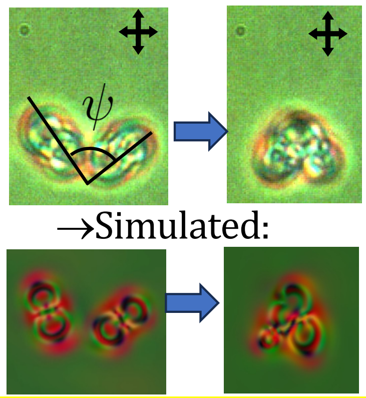




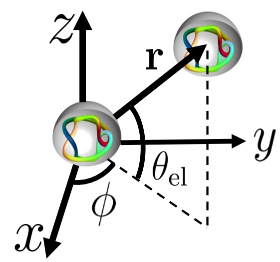
https://www.newsweek.com/near-limitless-fusion-energy-closer-nuclear-breakthrough-1895556#slideshow/2385676

Inter-knot interaction energy driving the fusion

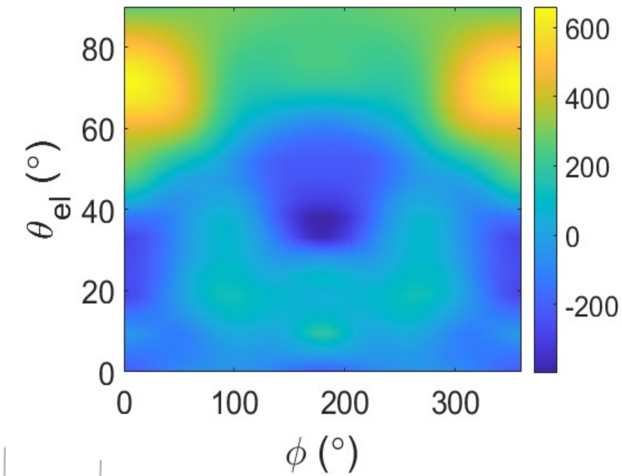




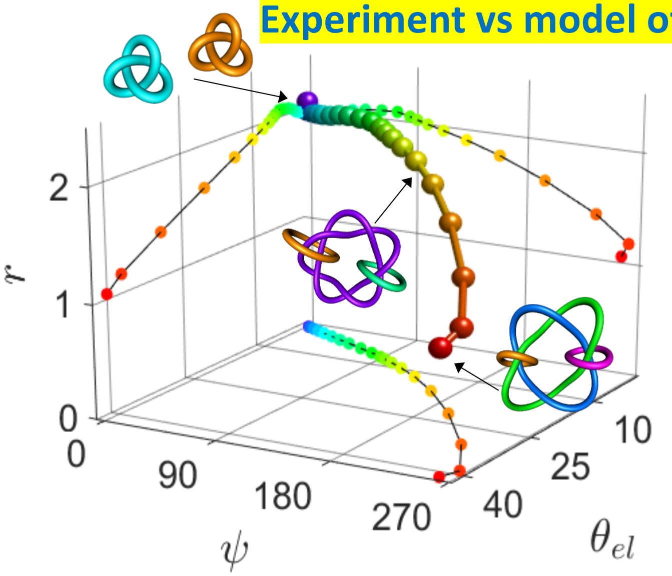
→Experiment geometry

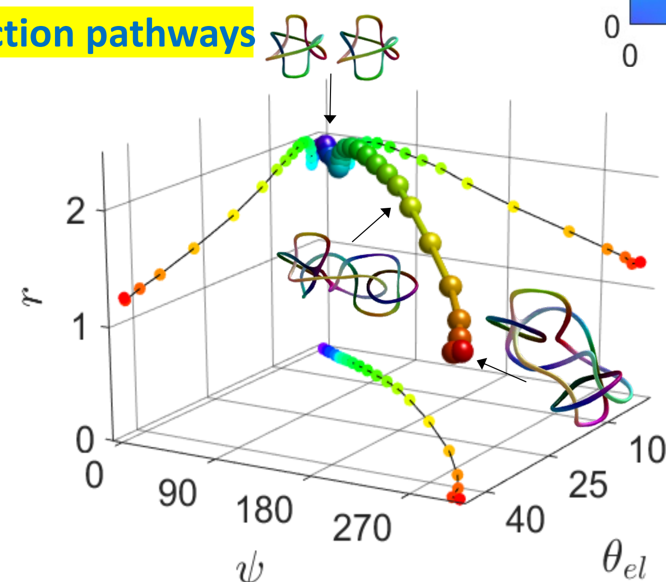


→Computed interaction energy



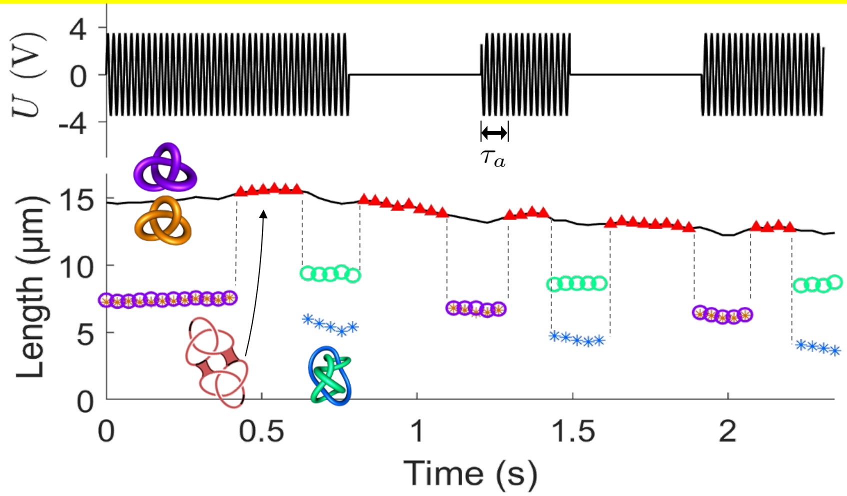
Experiment vs model of reconnection pathways



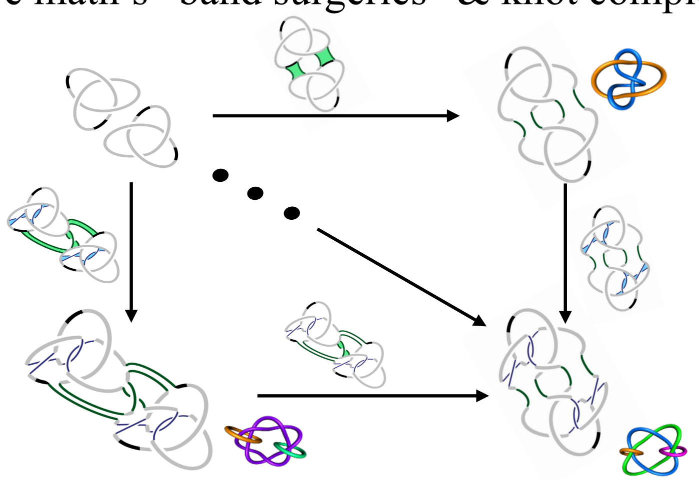


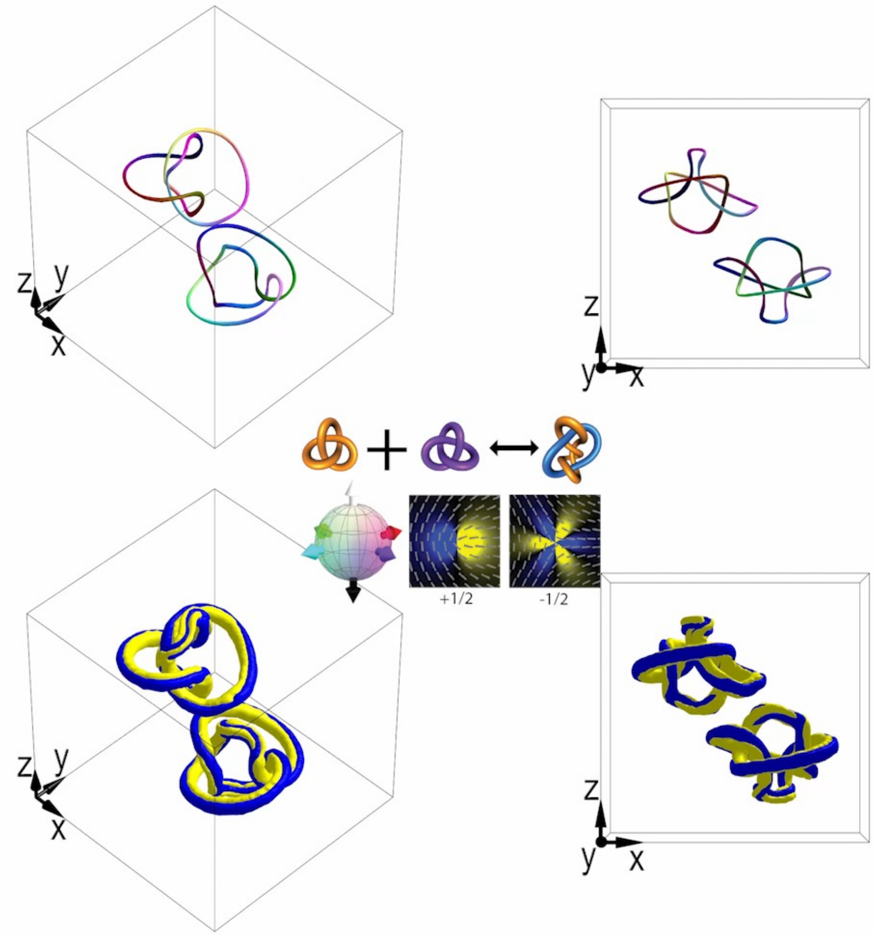
→Q preserving vortex knot reconnections & corresponding invariants

Reversible “double” reconnections & knot switching



Pure math’s “band surgeries” & knot complexity





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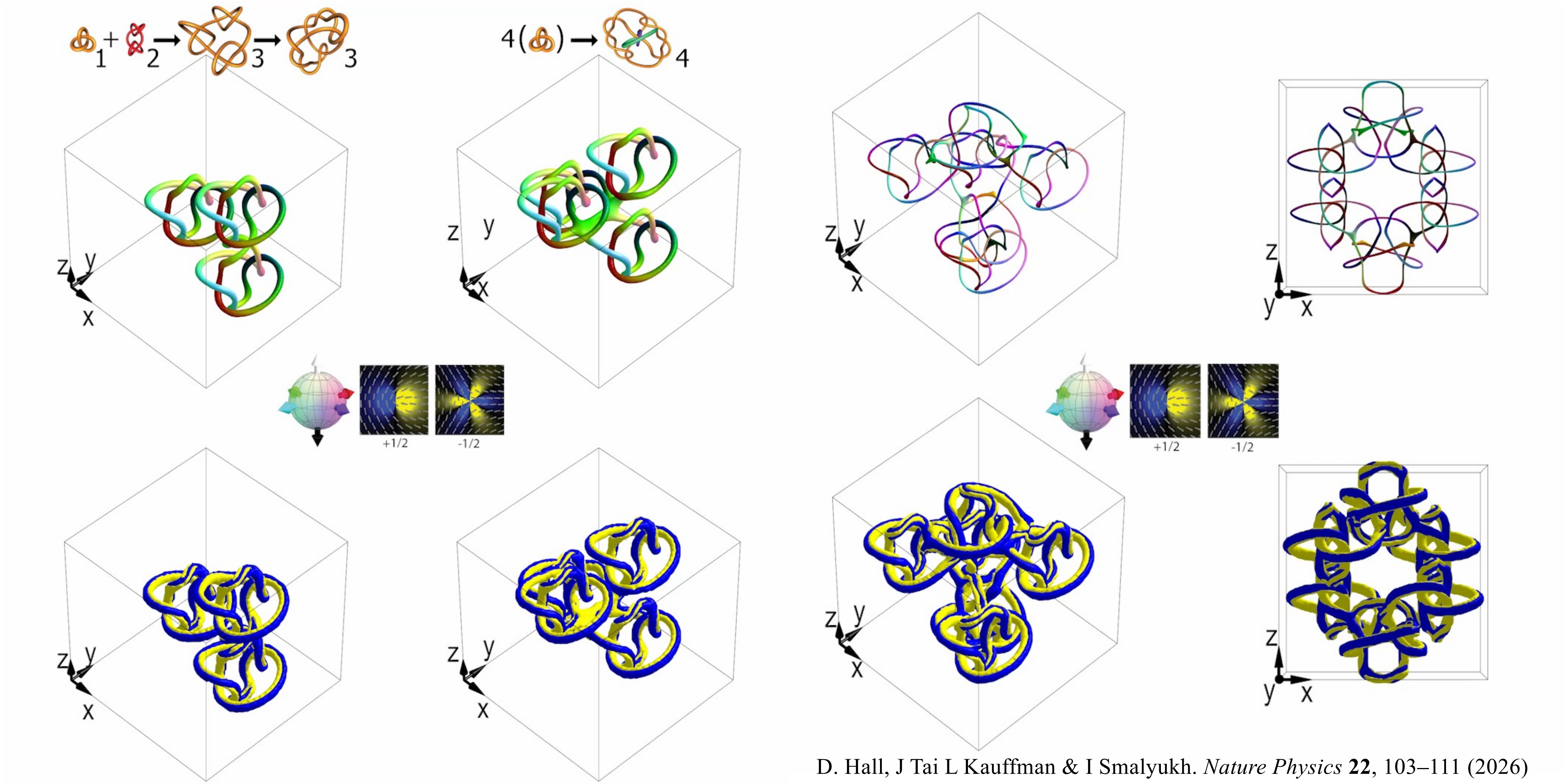
More & more complex knots created via electro-fusion



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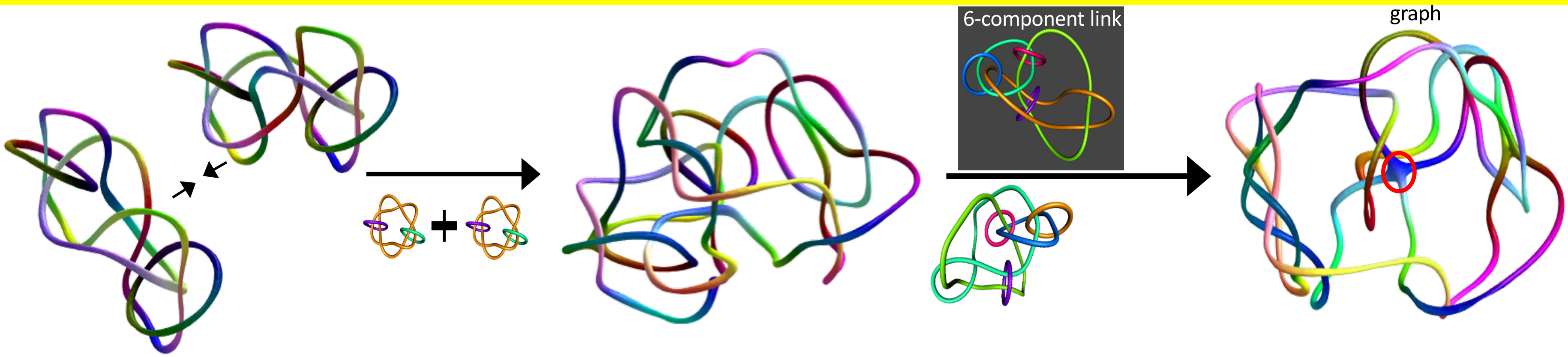
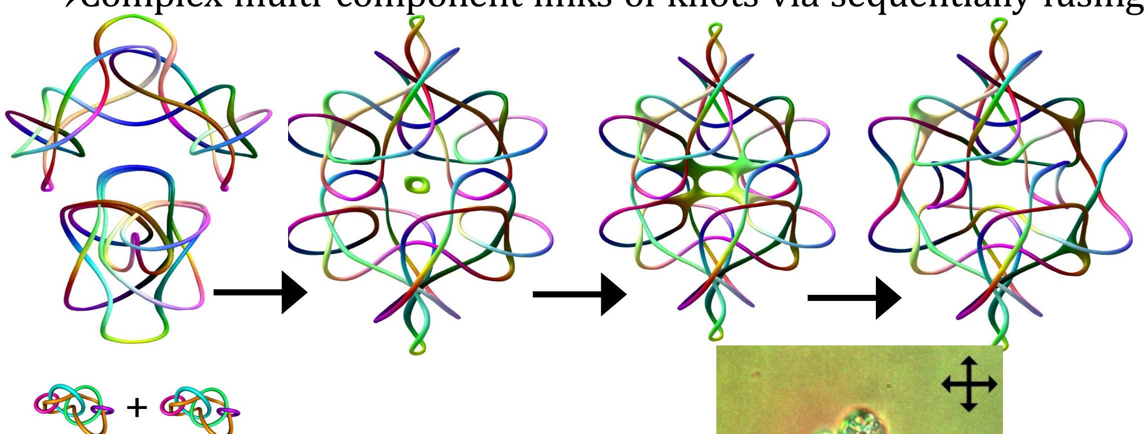
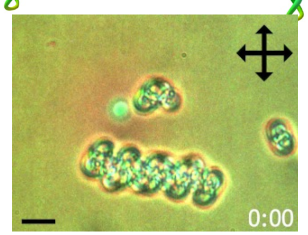
Complex knots & graphs through fusion

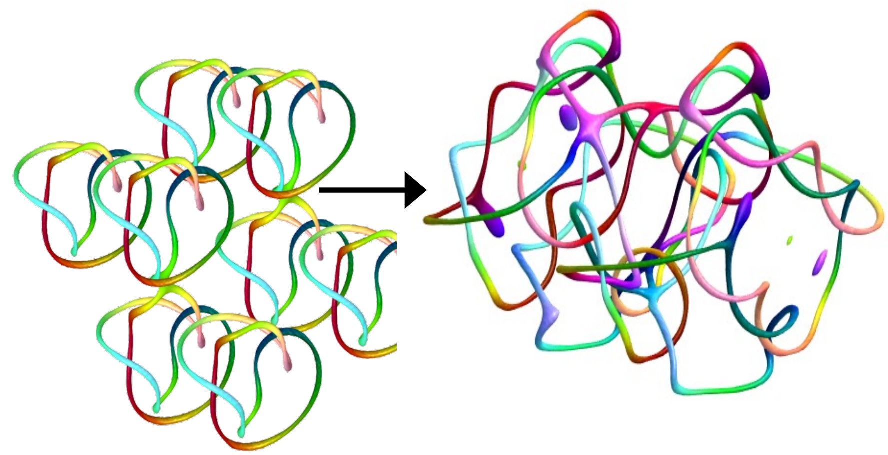


→Complex multi-component links of knots via sequentially fusing simple ones





→Fusion also guided by laser tweezers



→Fusion from an array

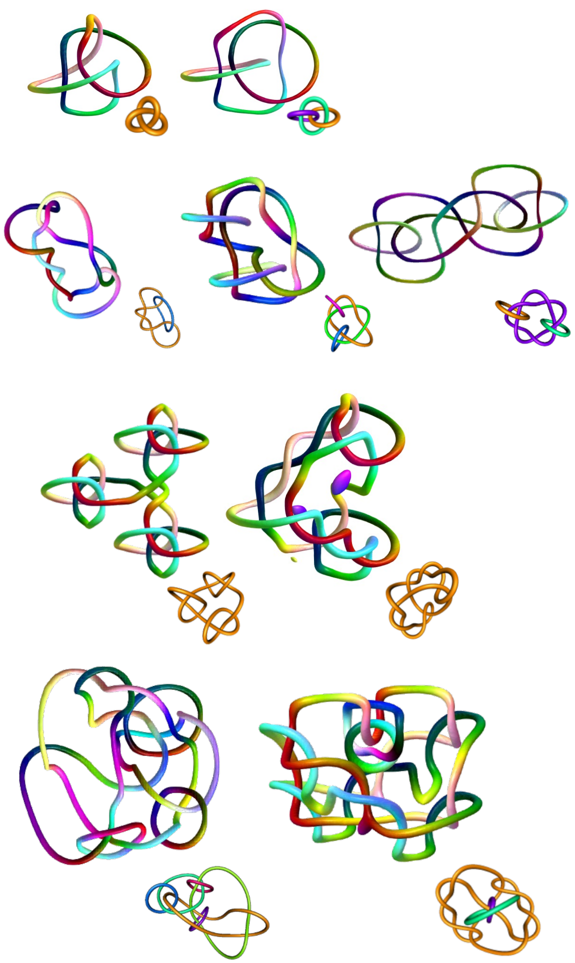
What are the “atomic” or “baryon” numbers?

1

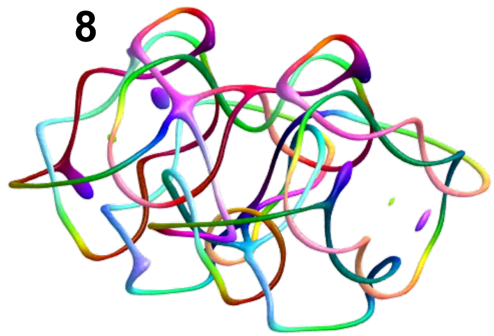
2

3

4

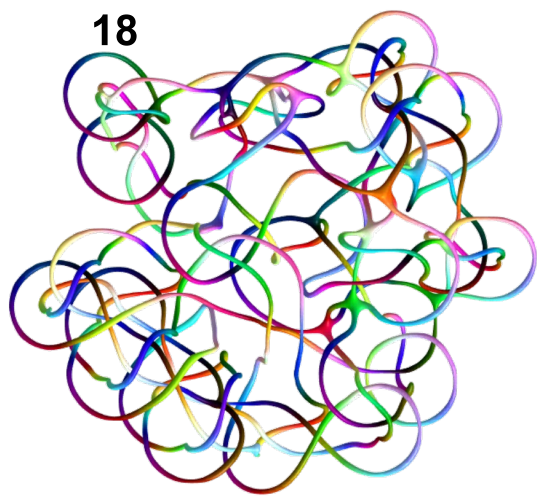


8



⋮

18



$$Q = \frac{1}{64\pi^2} \int_{\mathbb{R}^3} d^3\mathbf{r}\epsilon^{ijk} A_i F_{jk},$$

$$F_{ij} = \epsilon_{abc} n^a \partial_i n^b \partial_j n^c, \quad F_{ij} = (\partial_i A_j - \partial_j A_i)/2,$$

→Particle-like Hopf solitons
→Vortex links & knots in helical axis field
→Energy-minimizing states

What happens to the knot chirality upon fusion

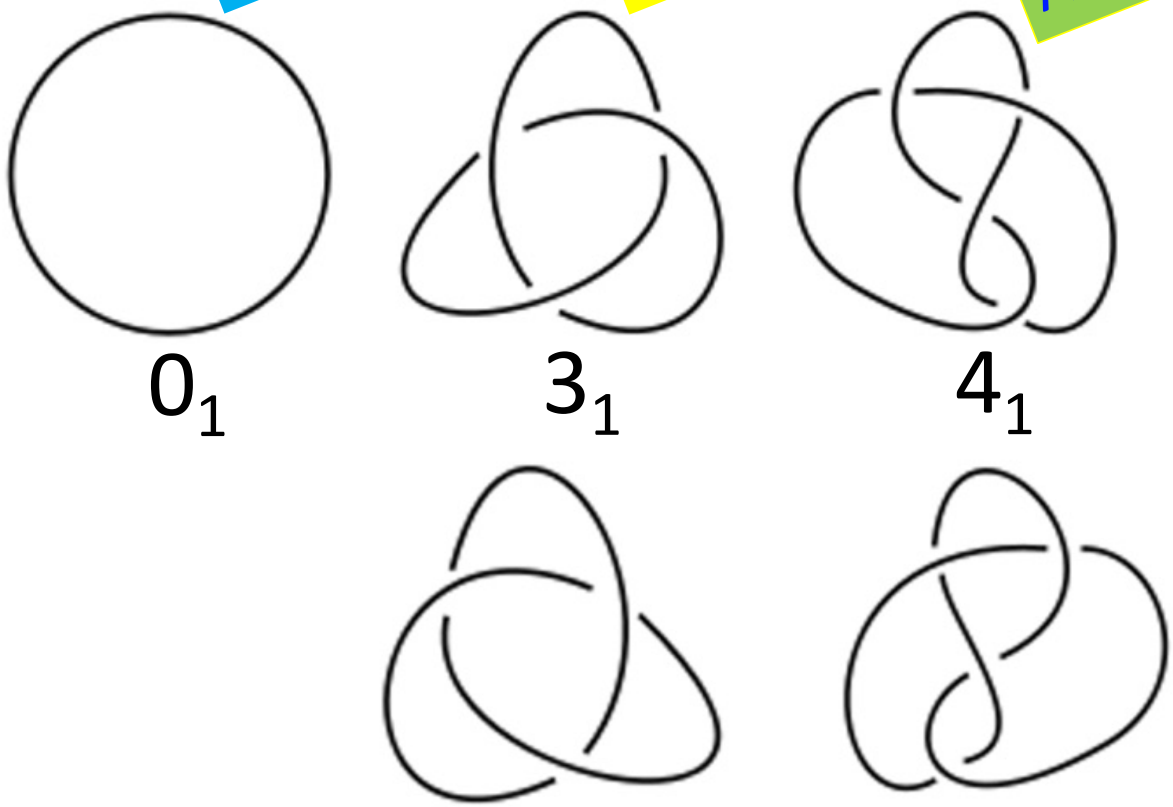
D. Hall, J Tai L Kauffman & I Smalyukh. *Nature Physics* **22**, 103–111 (2026)

Chirality and Knots (3 simplest knots)

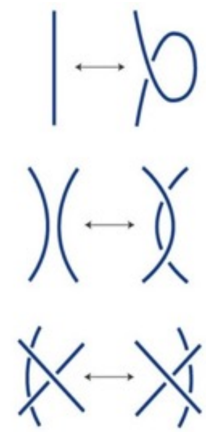
Achiral

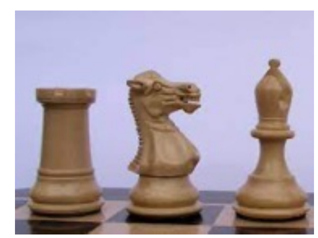
Chiral

Amphicheiral

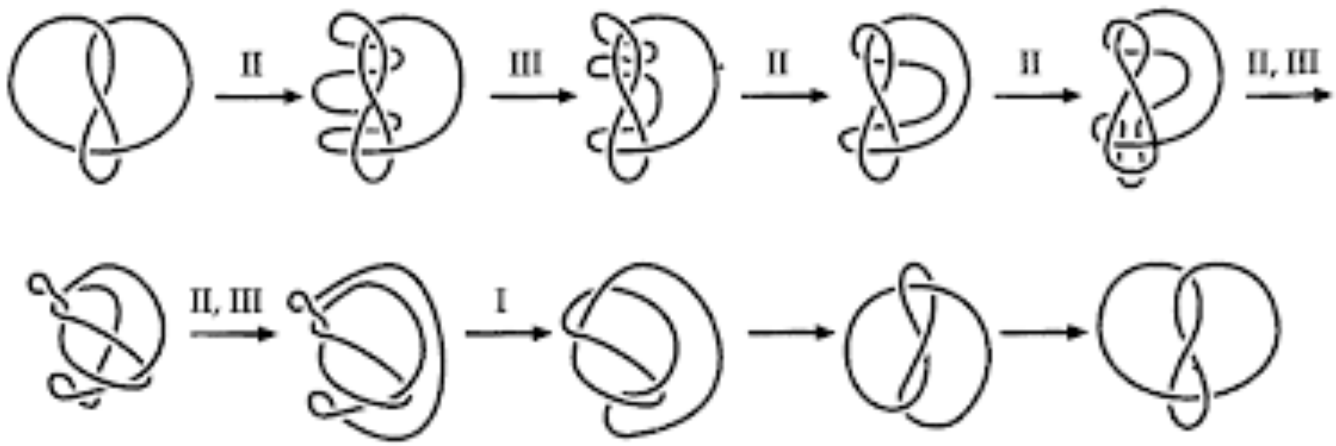


Reidemeister moves are chess moves allowed by topologists.





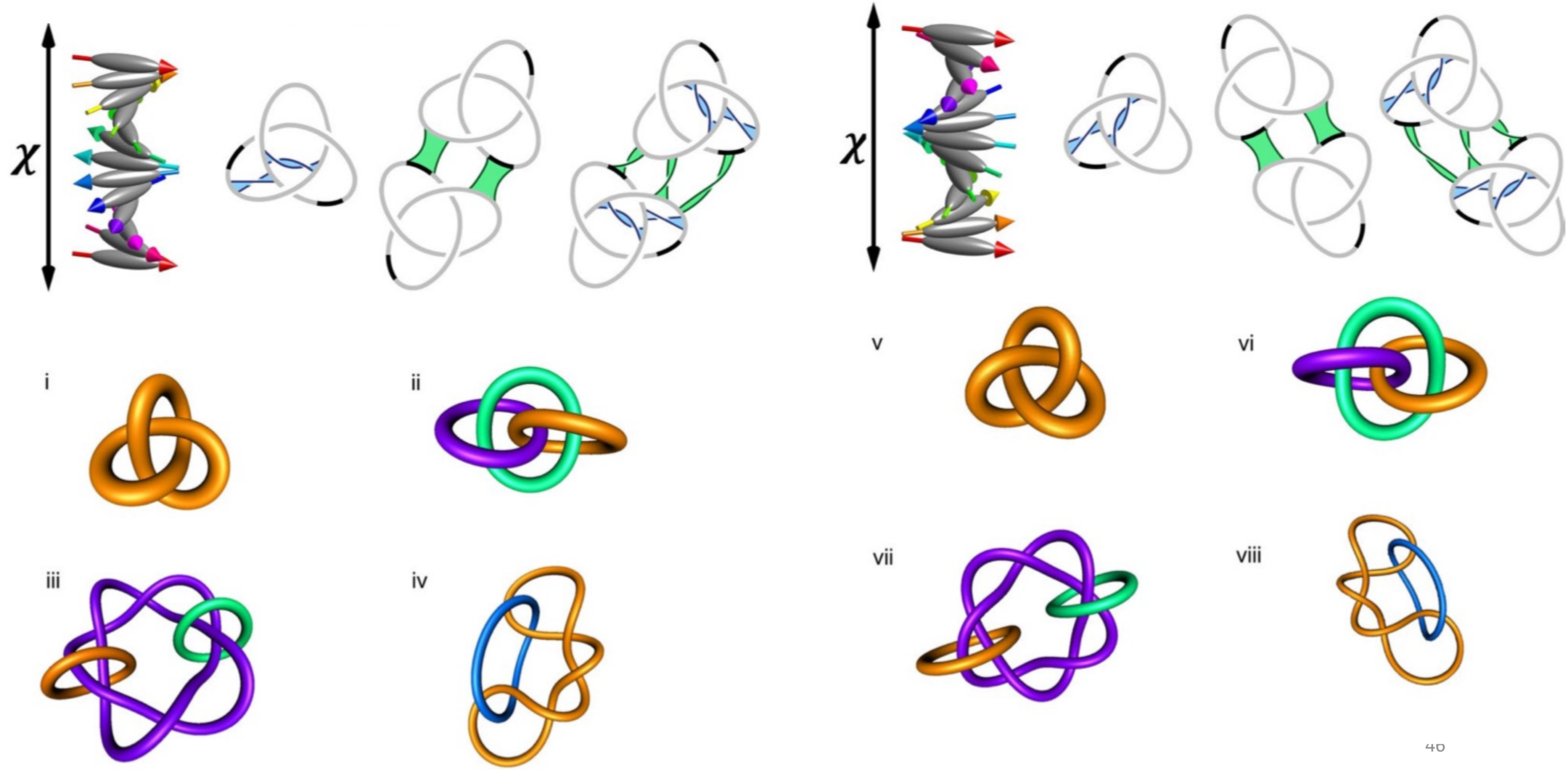
Conversion of amphicheiral figure 8 knot to enantiomorph Reidemeister move sequence.



In this sequence of moves, the knot never becomes achiral.

But it could. S_4 symmetric.

Chirality of vortex knots & their fusion?



Summary & conclusions

→Vortex knots in chiral LCs as **“particles”**

→Crystalline self-assemblies of knot solitons

→Electro-fusion & -fission

→Cosmology-inspired steering & knotting of light beams

→Vortex metaatoms or topological metamaterial design principle?

Thank you !!!