Entanglement & dynamics of LC vortices

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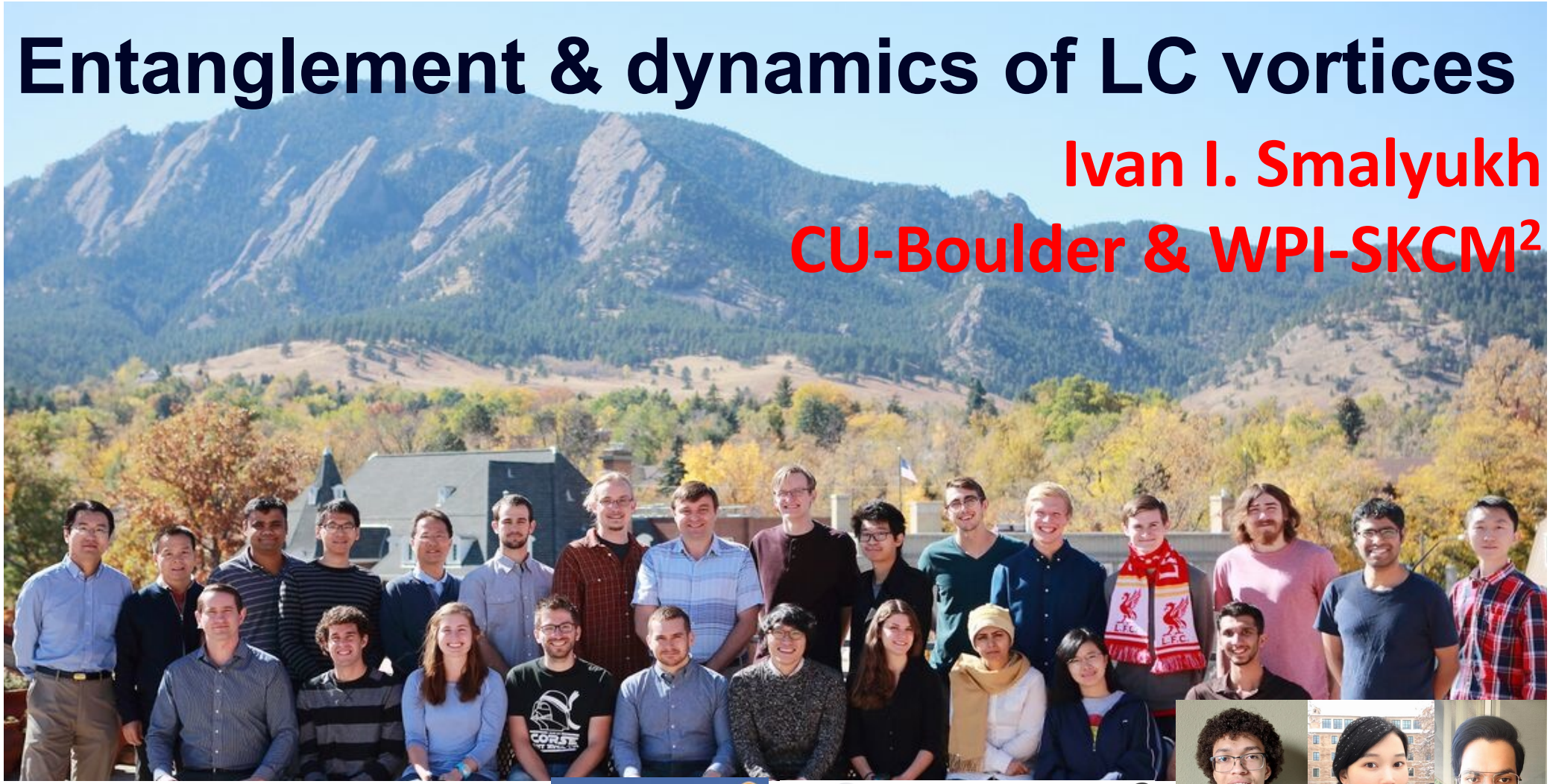

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Surfaces, topological defects & topological solitons

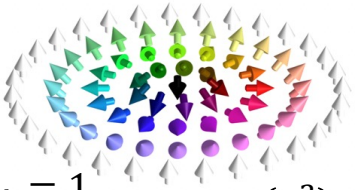
Surfaces

genus 1

genus 0

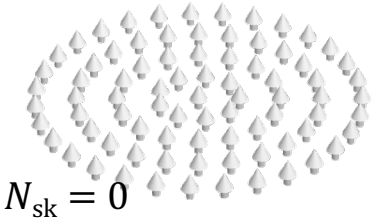
https://en.wikipedia.org/wiki/Topology

Nonsingular fields

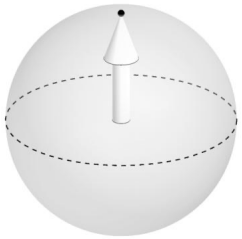


$N_{\mathrm{sk}} = 1$

$\neq$ $\pi_2(\mathbb{S}^2) = \mathbb{Z}$



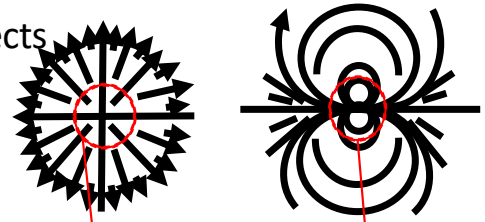
$N_{\mathrm{sk}} = 0$



Order-parameter space: $\mathbb{S}^2$

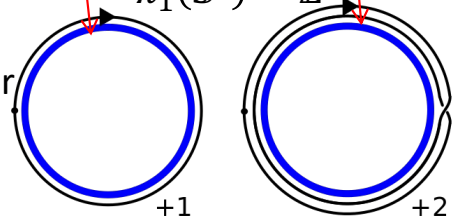
Singular defects

2D defects



$\pi_1(\mathbb{S}^1) = \mathbb{Z}$

Order-parameter space: $\mathbb{S}^1$

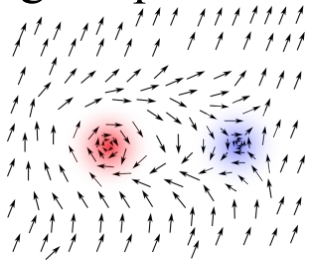


	π_1	π_2	π_3
$\mathbb{S}^0$	0	0	0
$\mathbb{S}^1$	$\mathbb{Z}$	0	0
$\mathbb{S}^2$	0	$\mathbb{Z}$	$\mathbb{Z}$
$\mathbb{S}^3$	0	0	$\mathbb{Z}$

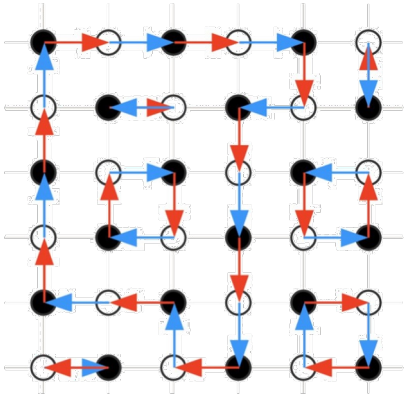
So much more:

→Topological insulators…

→Topological phases

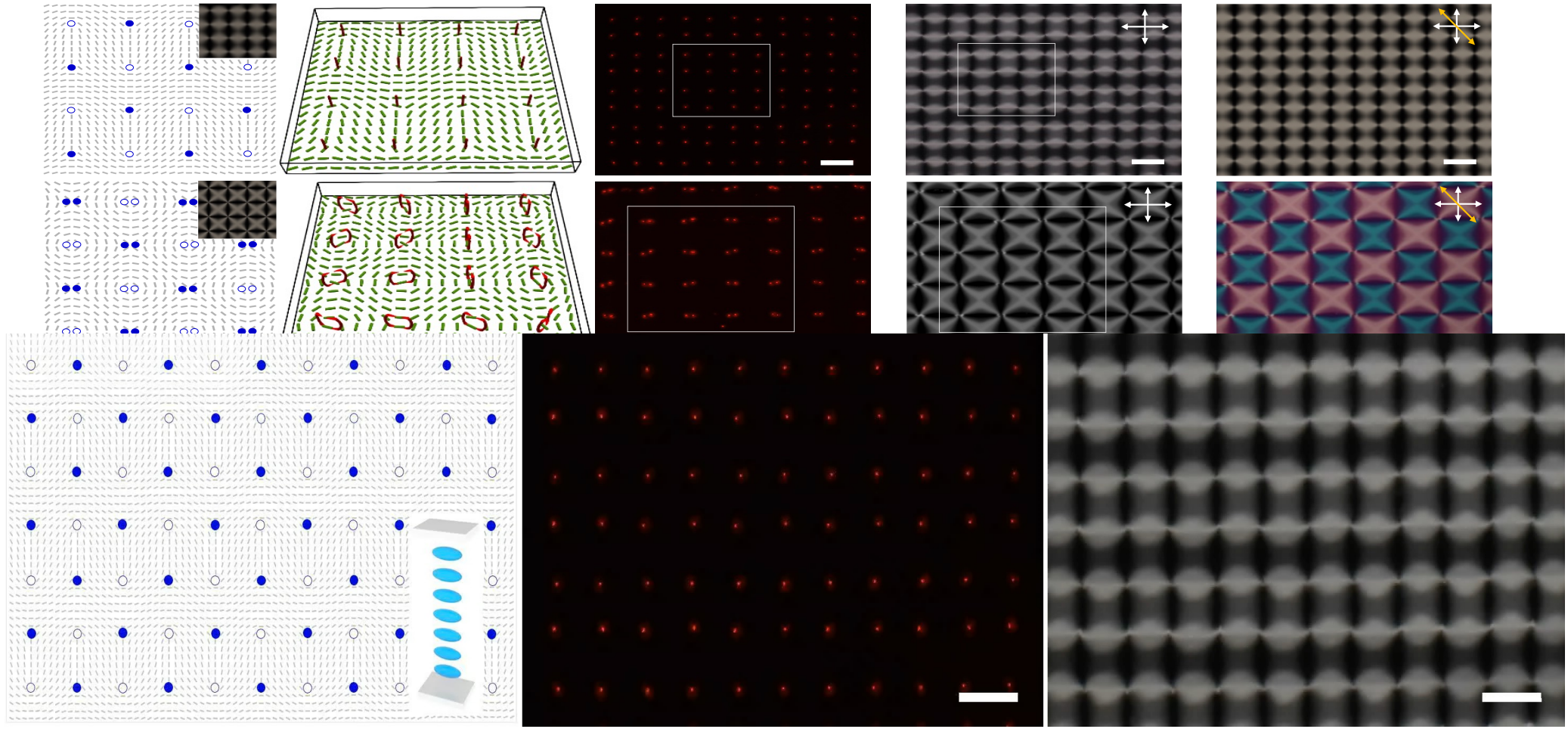


→Topological order…



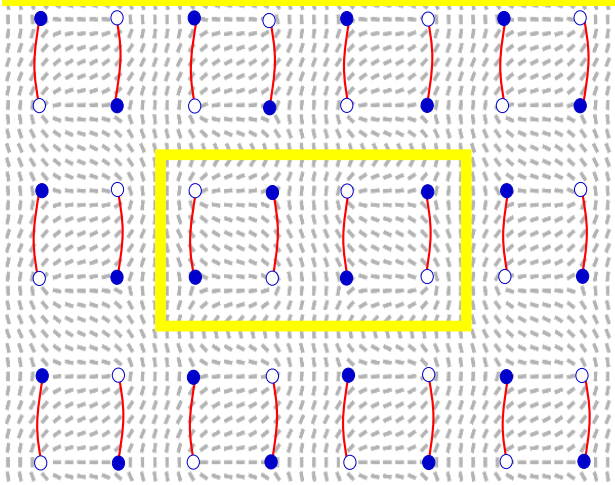
Topological invariants/sectors, properties preserved under **continuous** deformations.

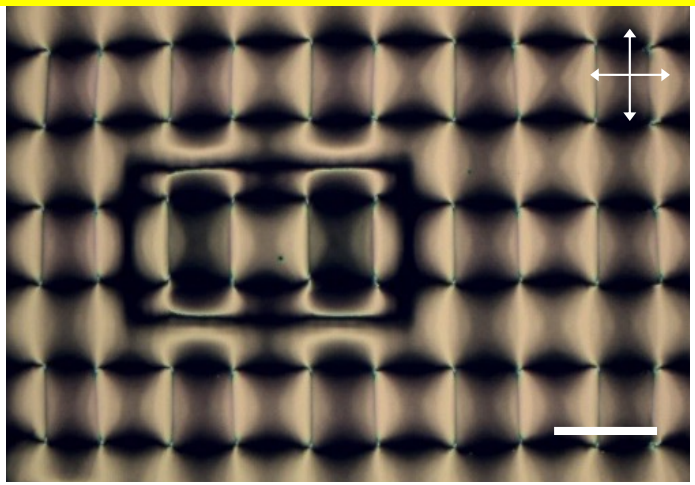
Crystals of nematic defects with different symmetries

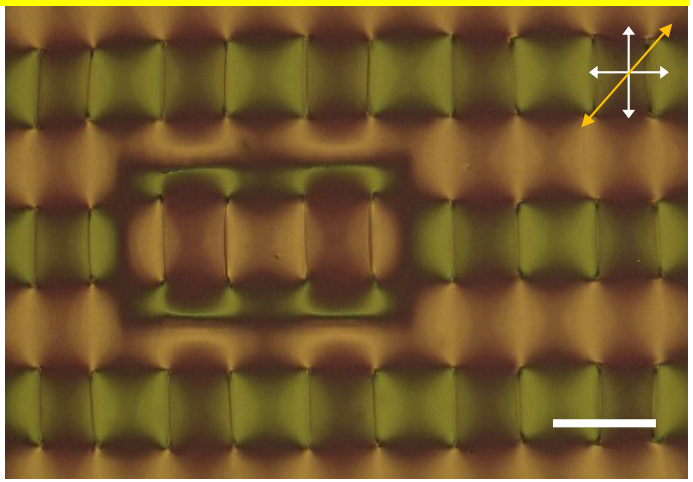


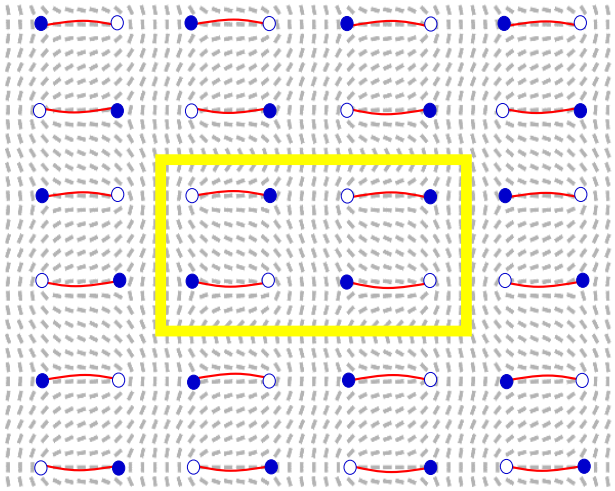
→Crystals of atom-like topological quasiparticles… conventional defect topology

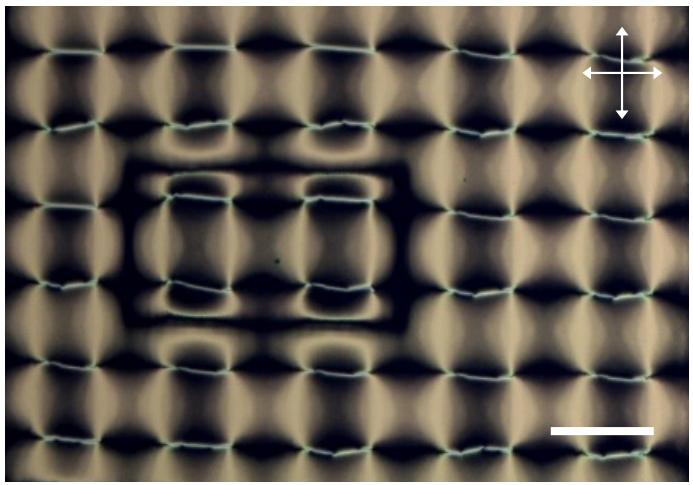
Crystal defects: grain boundaries, substitutional defects…

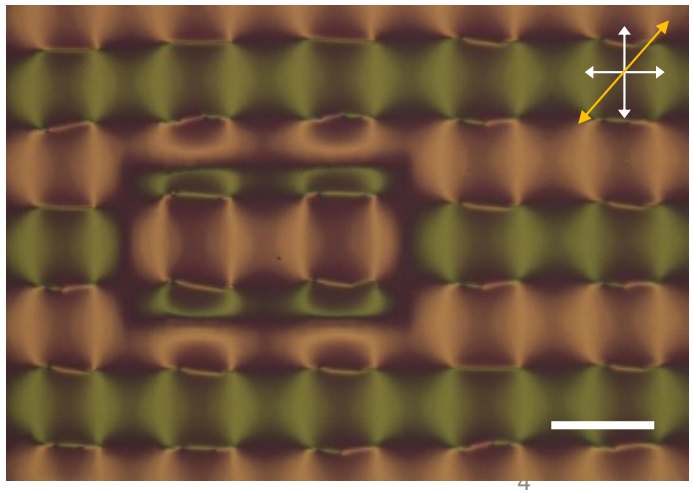












→Higher level defects in positional order… still conventional defect topology, winding number conservation

Classical Landau Theory of Phase Transitions & LCs

transition (without a jump in the energy) at a single point in the p-T plane. The p-T diagram near this point has in the simplest case the form shown in Fig. 1. Here I denotes the region of the more symmetrical phase. II*a* and II*b* are two different phases with the same symmetry, which is that of a subgroup of I. At the point O all the three phases become identical.

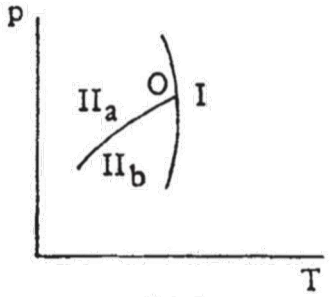


FIG. 1.

It can be shown that the transition crystal-liquid belongs to this class. No Curie point line can therefore exist for melting, and the only possible type of continuous transition crystal-liquid is that shown in Fig. 1, where in this case I refers to the liquid and II to the crystalline state.

L. LANDAU.
Ukrainian Physico-Technical Institute,
Kharkov.
Sept. 7.

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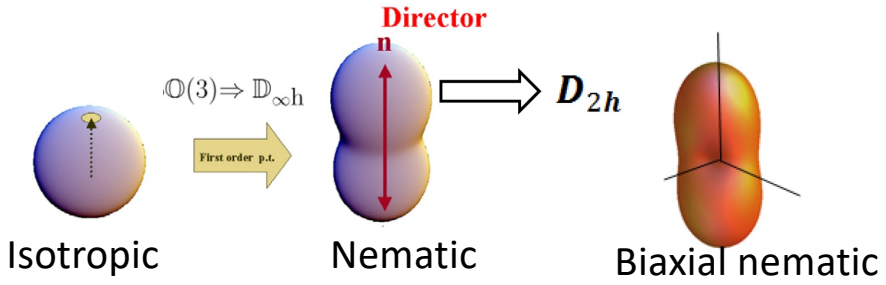
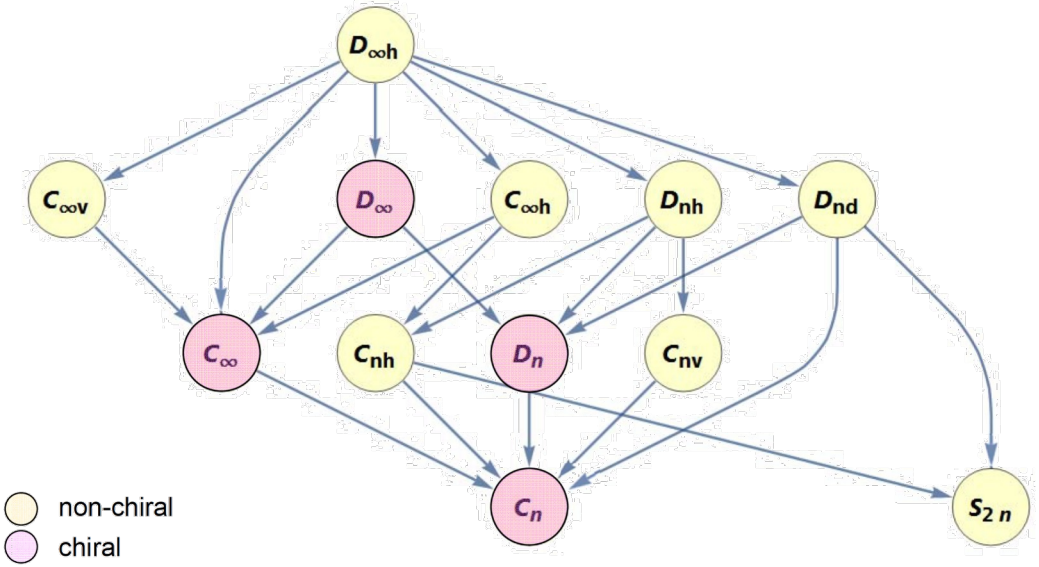
→90 years of Landau theory

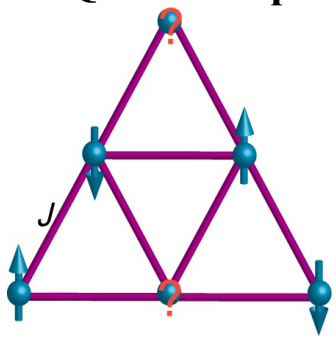


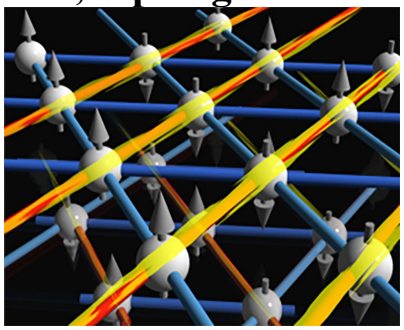
→Order parameters describing phase transitions
→Symmetry breaking applied to LC phases, crystals…



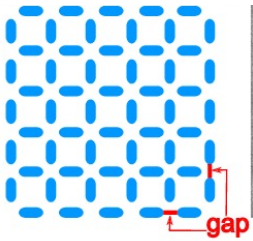
Phase transitions: beyond just local symmetry breaking

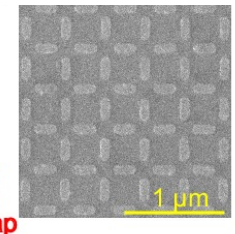
→Quantum spin liquids, topological order

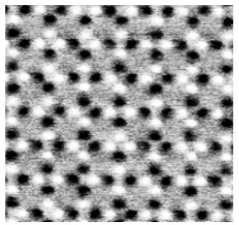


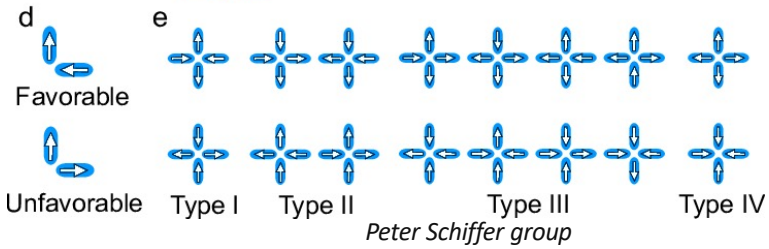


→Multiplicity of "entangled states"
→Energetic degeneracy of different states
→Topological order in quantum & classical systems
→Artificial "spin ice" model systems



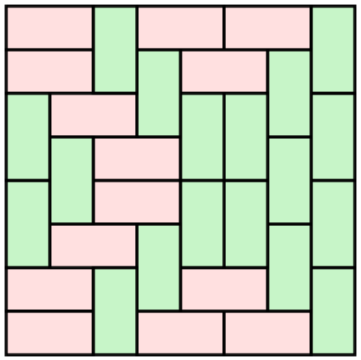


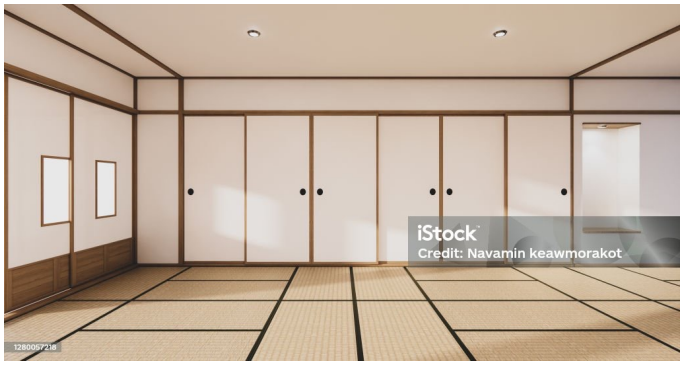




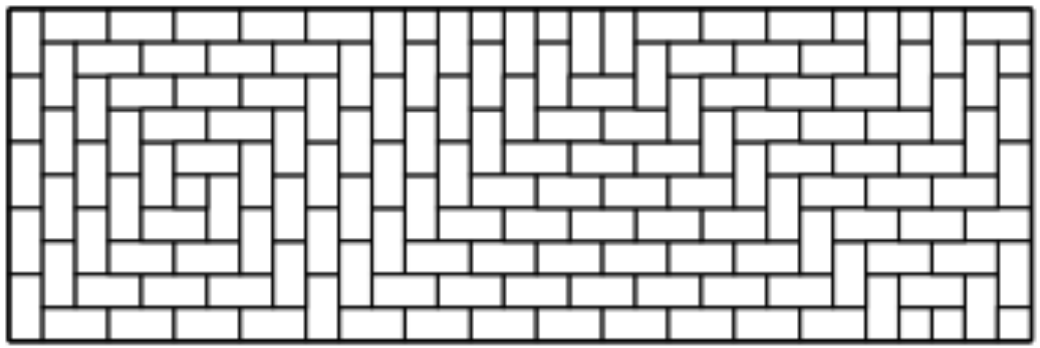
Peter Schiffer group

→Domino, tatami & mathematical "Dimer Model"
→The simplest long-range interactions:



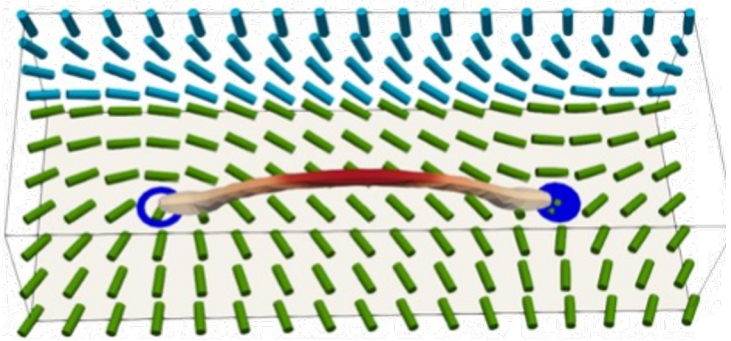
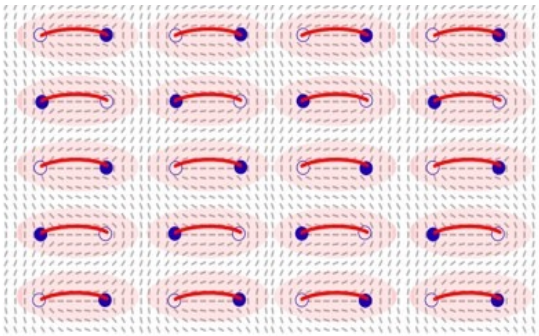
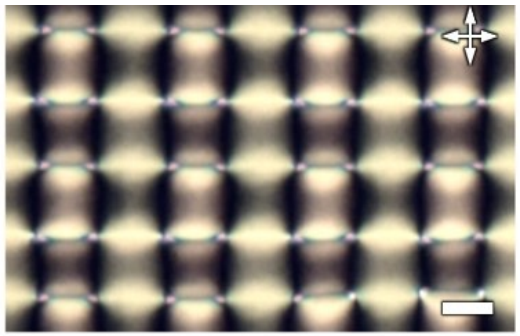


→Many ways to tile domino or tatami
→Cannot change the state of just one "domino"!!!
→The states are "entangled", inter-dependent

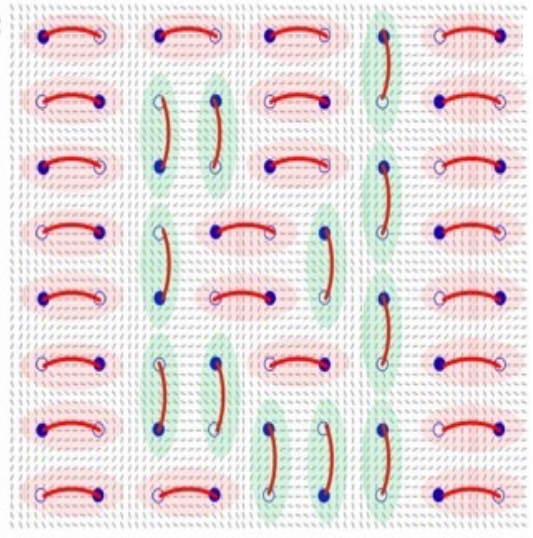
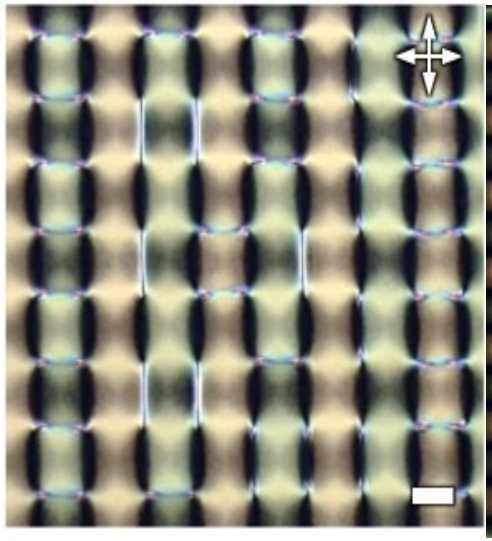
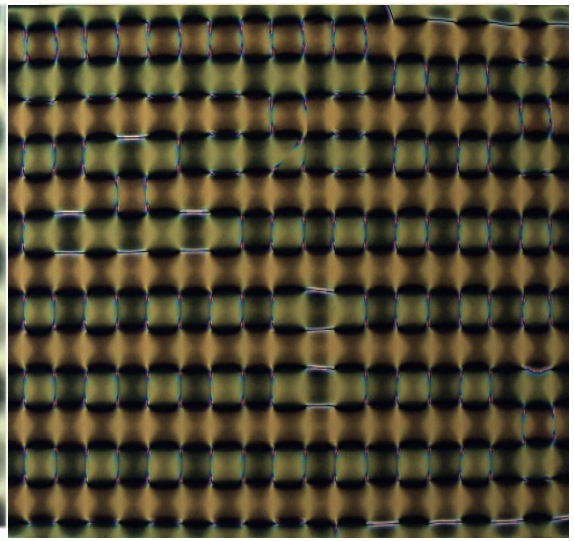
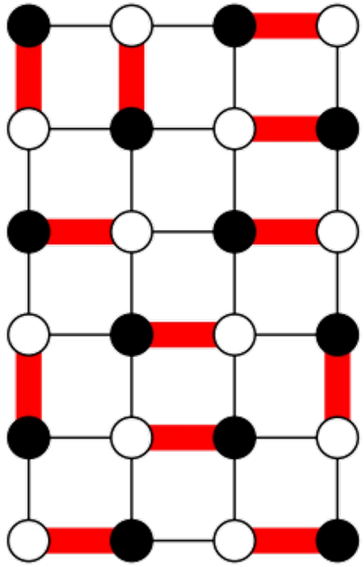


Dimer model & nematic combinatorial vortex lattices

→Hybrid homeotropic & patterned tangential BCs: “ordered” lines connecting pinning sites

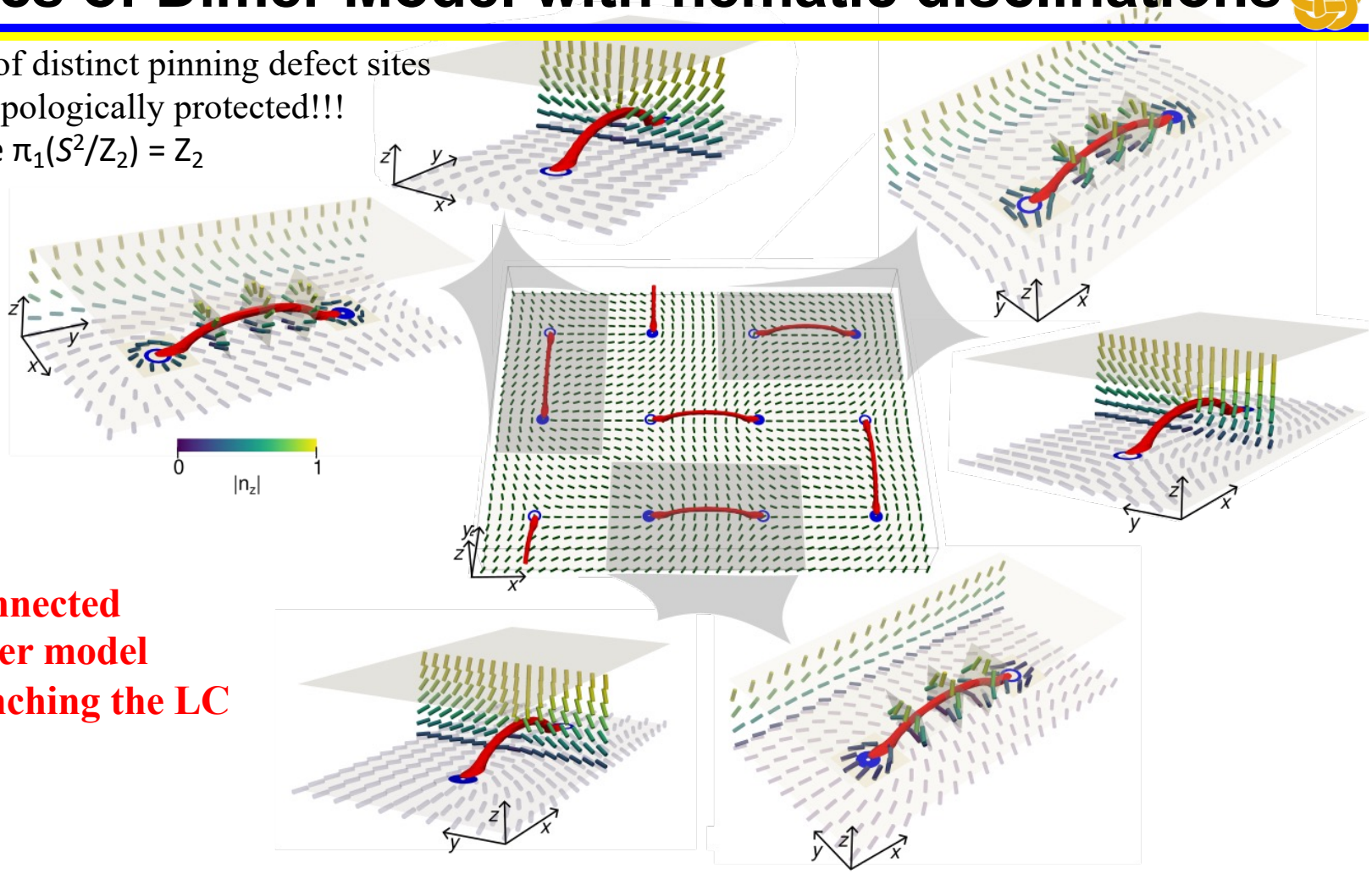
→Multiplicity of possible states after each order-disorder transition

→Number of configurations scales exponentially with lattice size

Combinatorics of Dimer Model with nematic disclinations

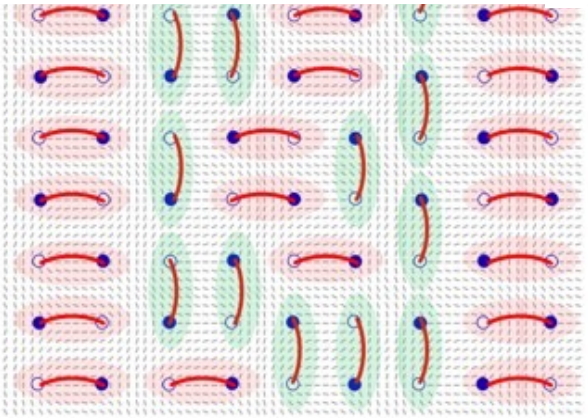
→Photopatterned lattice of distinct pinning defect sites
→Dimer arrangements topologically protected!!!
→In the bulk vortices are $\pi_1(S^2/Z_2) = Z_2$

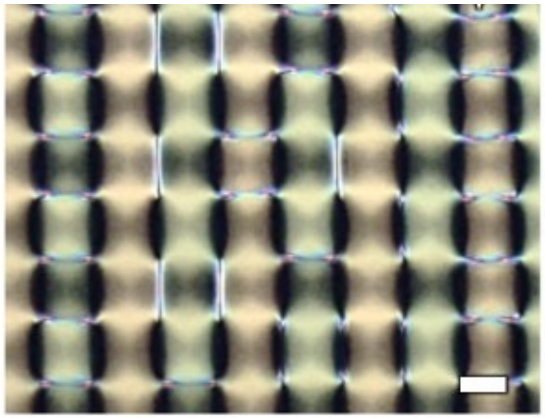


→All pinning sites connected
→Different dimer cover model states on melting/quenching the LC

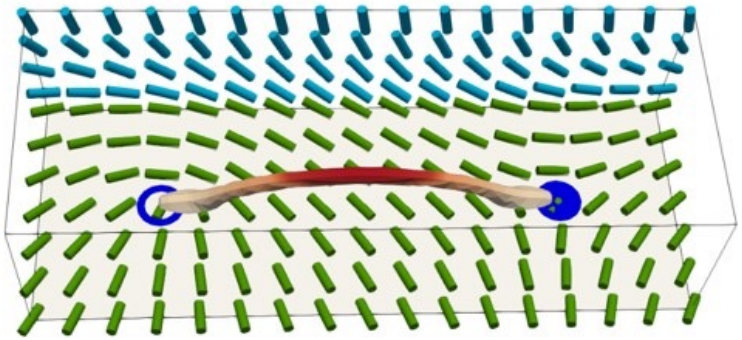
C. Meng, J. Wu, Z. Kos, J. Dunkel, C. Nisoli & I.I. Smalyukh, *Phys. Rev. X* **15**, 021084 (2025)

Topological order in lattices of topological defects



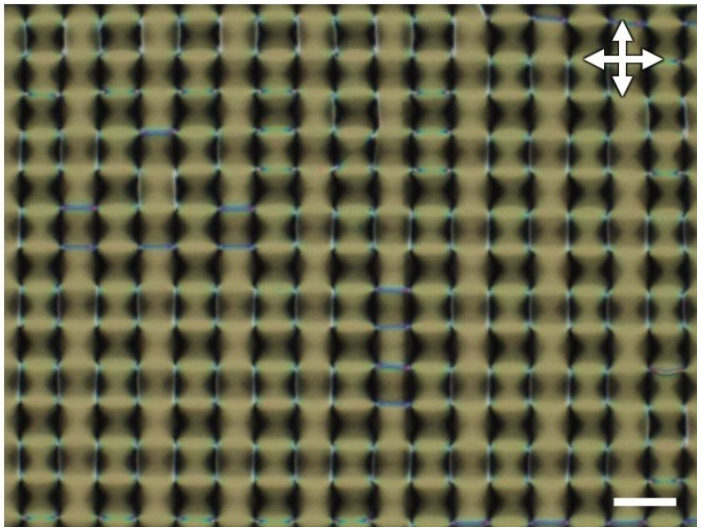


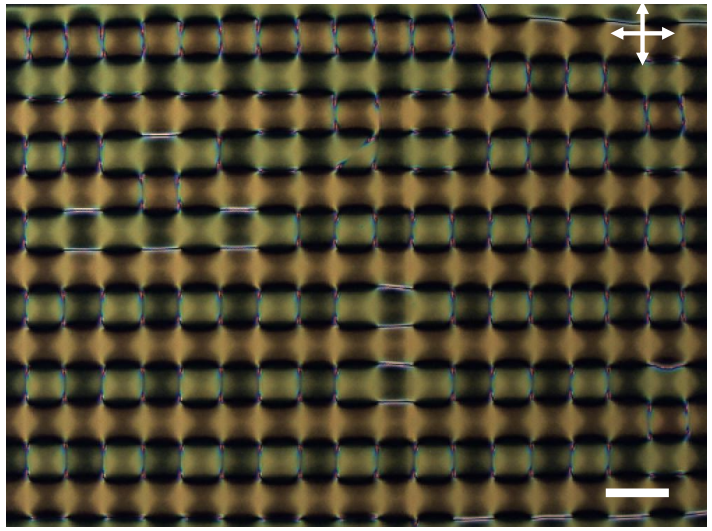
→A crystals of vortex dimers is one of many states

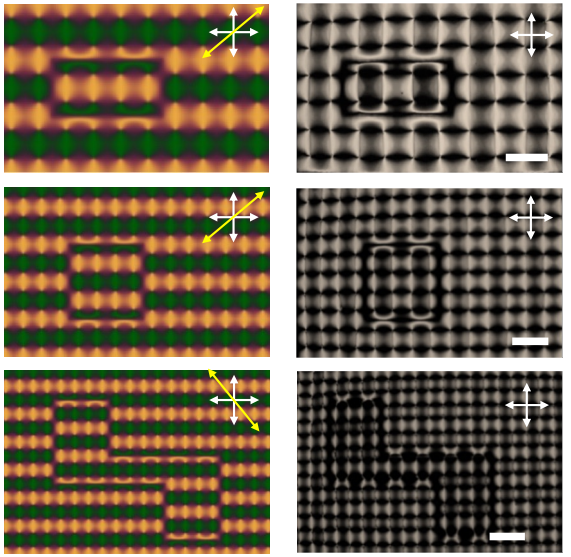


→new topological/entangled "protected" state after each melting/quenching

→number of configurations scales exponentially with lattice size

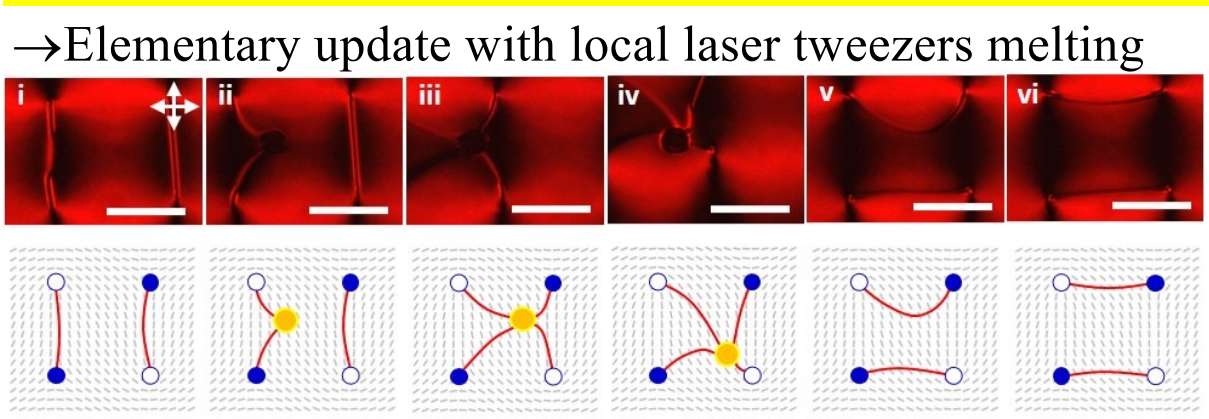


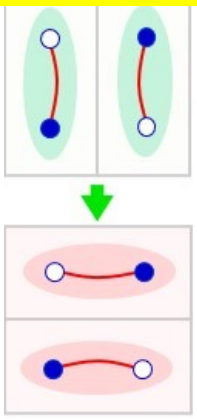


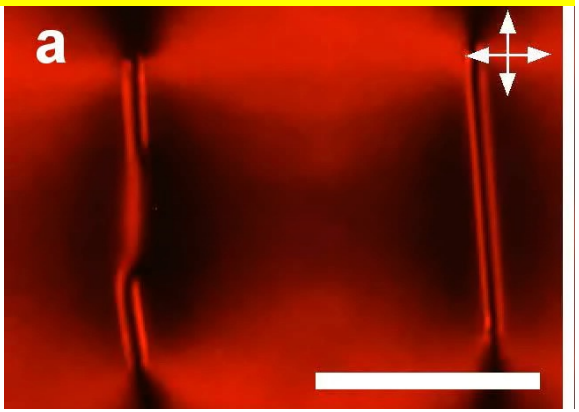


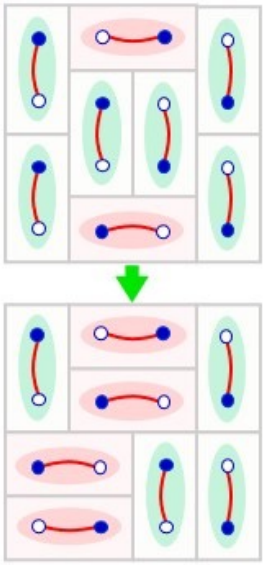
Updating Dimer Cover Model states with laser tweezers

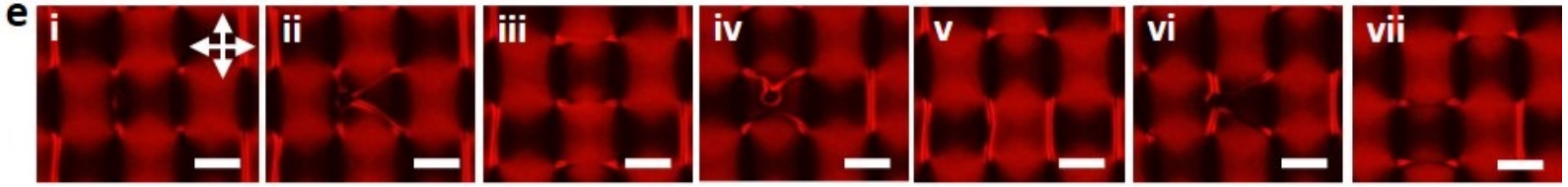
→Elementary update with local laser tweezers melting

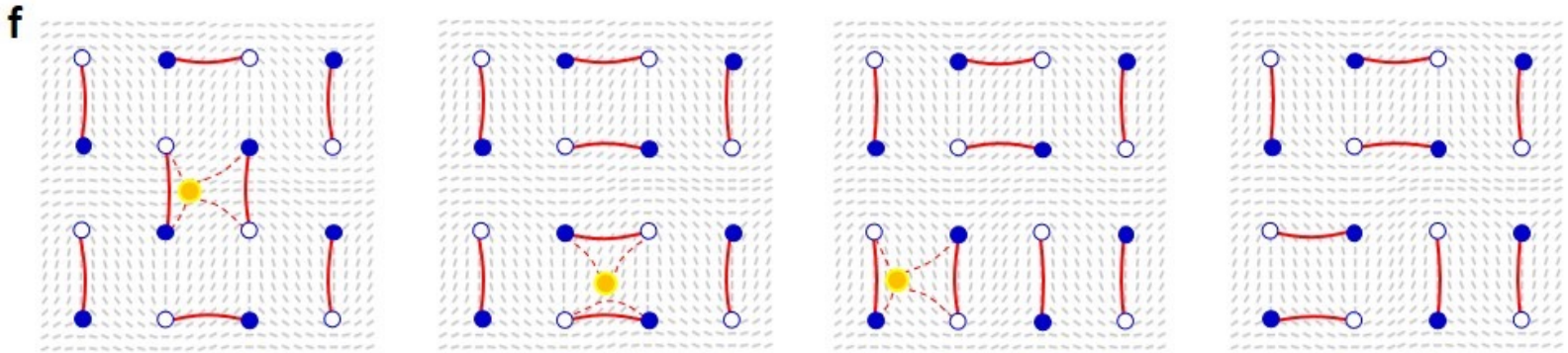












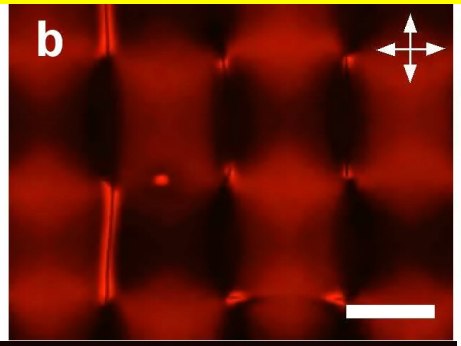
C. Meng, J. Wu, Z. Kos, J. Dunkel, C. Nisoli & I.I. Smalyukh, *Phys. Rev. X* **15**, 021084 (2025)

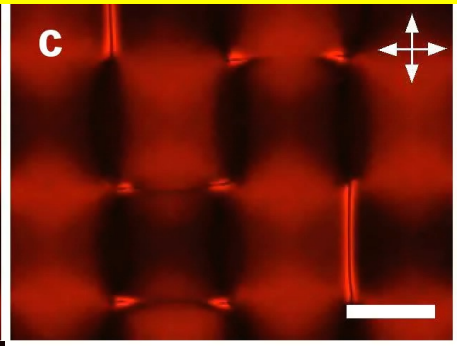
More complex updates in combinatorial vortex lattices

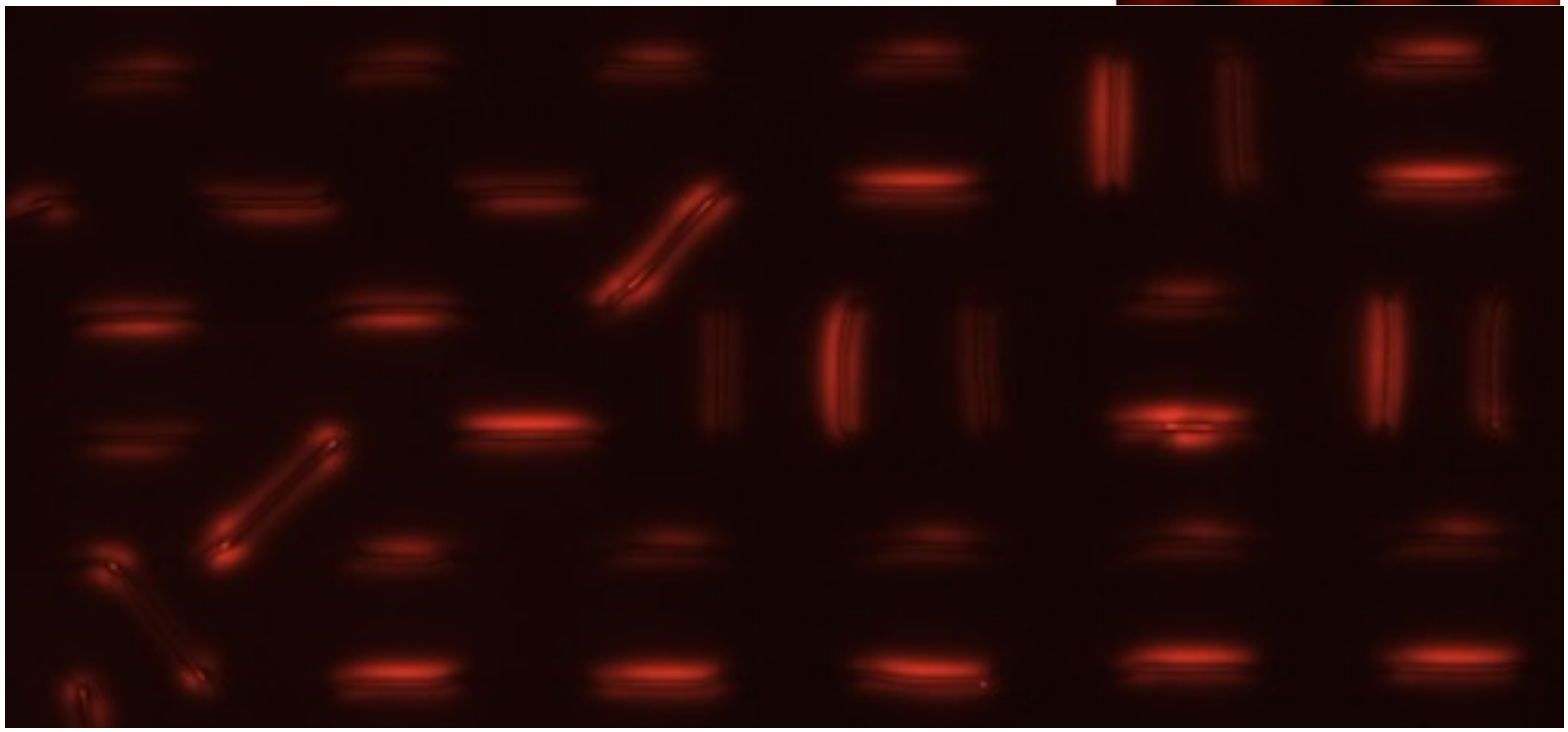
→Tweezers assisted “topological surgery” reconnections

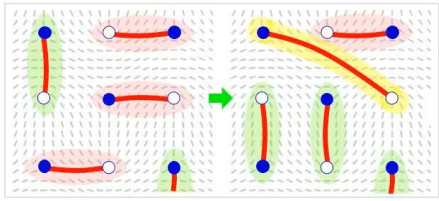
→Disclinations along diagonals of the lattice can emerge!

→What special about it?

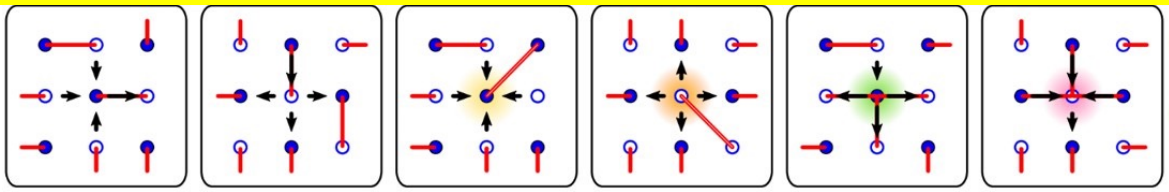


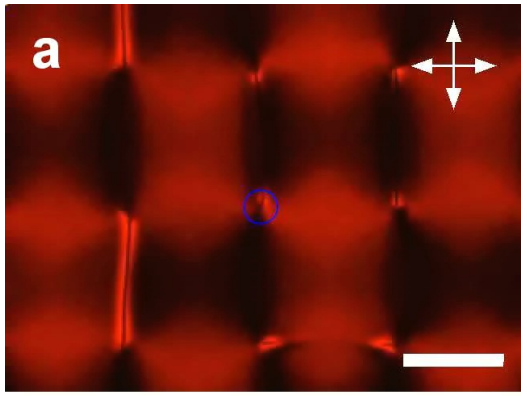




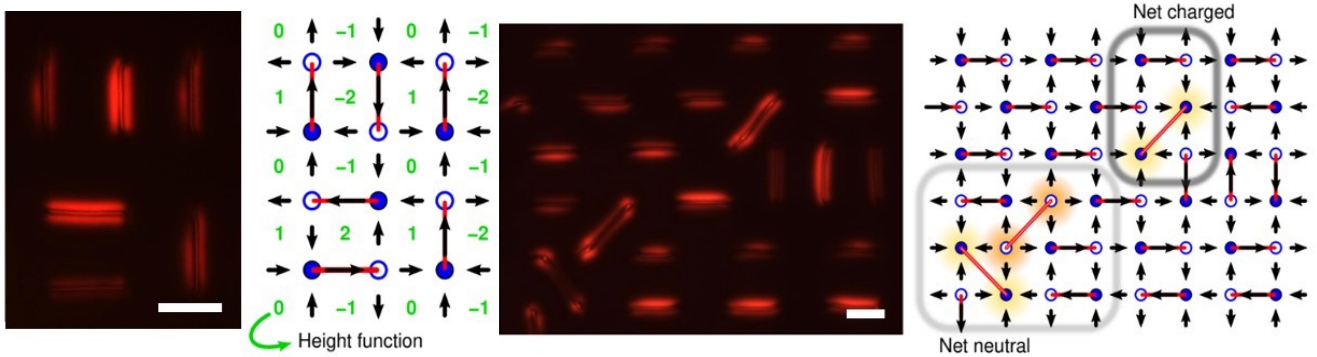


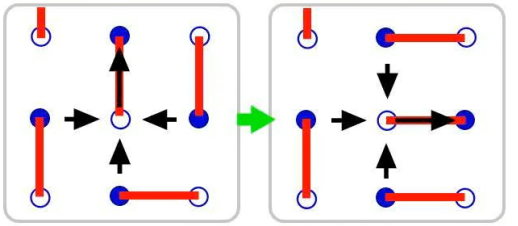
Emergent higher order dimer point defects & Dirac strings



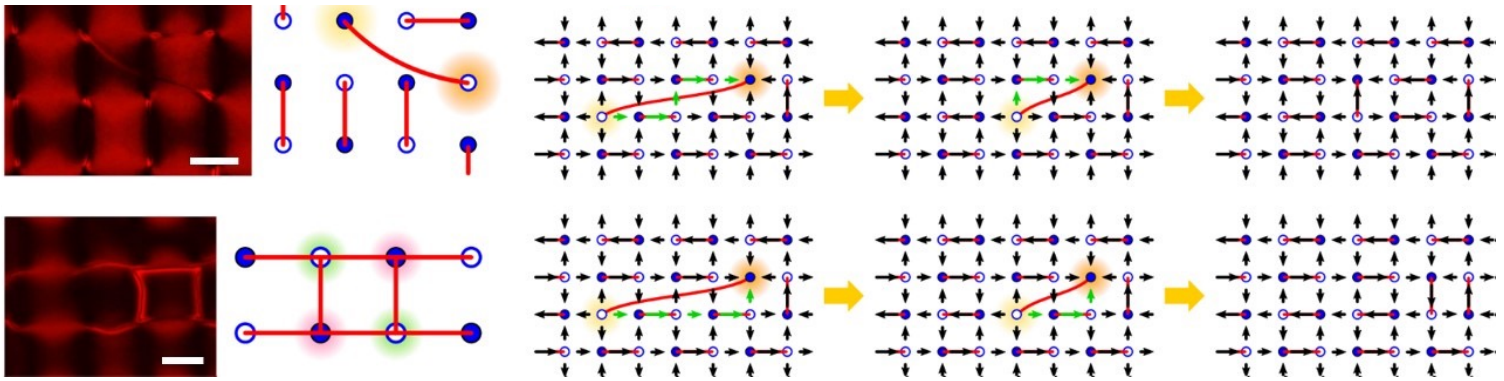


Conservation of charges of emergent higher-order defects in the lattices



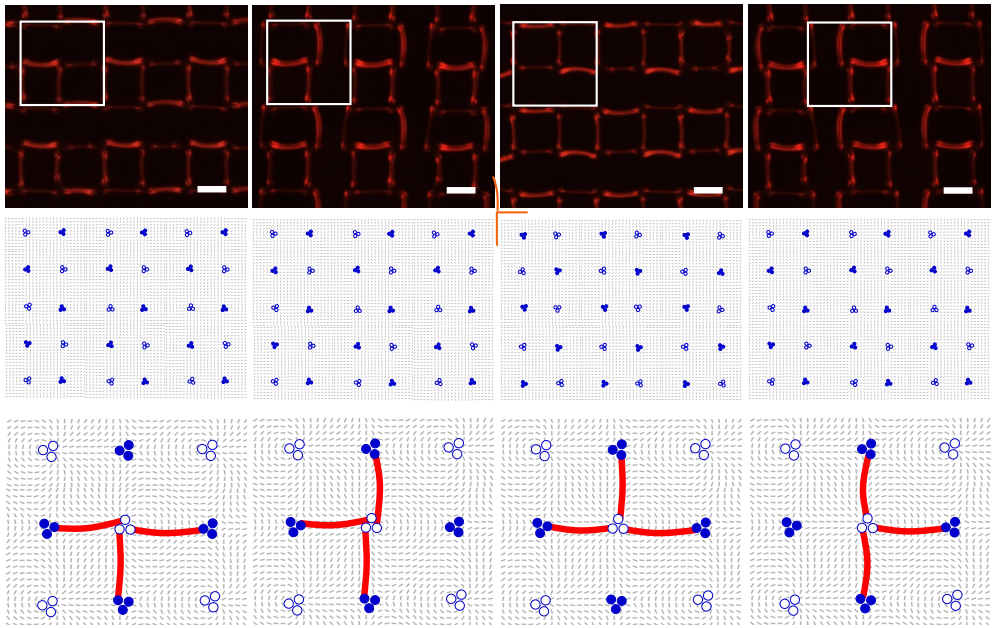


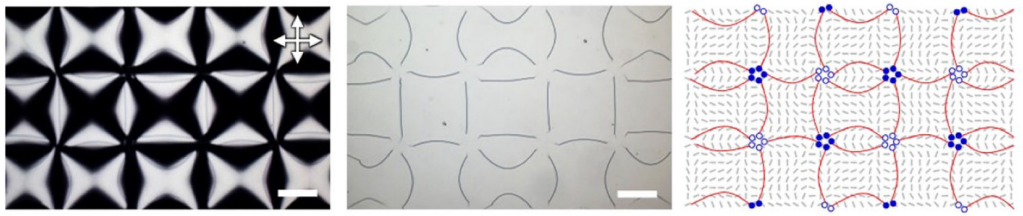
Dirac string analogs & annihilation of point defects



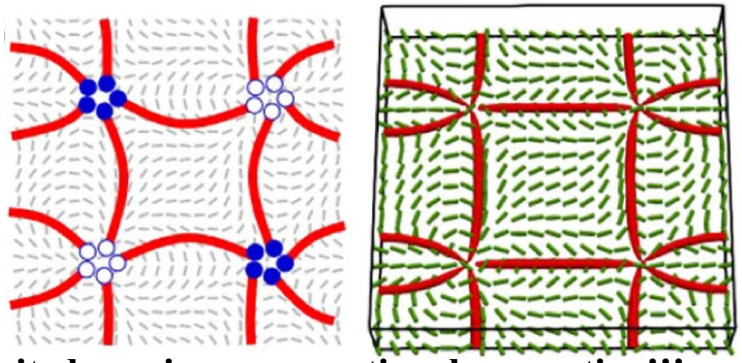
Engineering via basis-lattice “frustration”

→+3/2 & -3/2 defect pinning sites on a square lattice, pining 3 bulk disclinations
→“Missing” disclination “bond/spin” per each site - degeneracy of states
→Topological order & clustered states





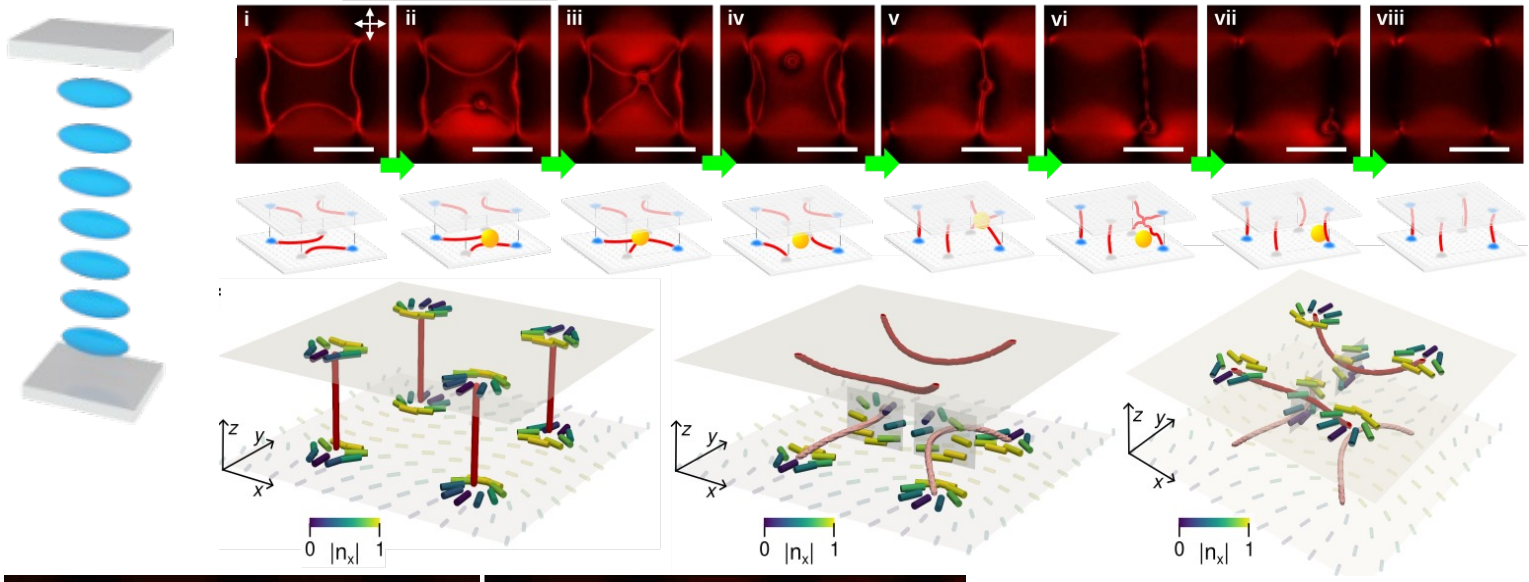
→+5/2 & -5/2 defect pinning sites on a square lattice, pining 5 disclinations
→Point symmetry of pentagonal pinning sites incompatible with crystal lattice
→Extra disclination “bond/spin” per each site
→Degeneracy of states & topological order



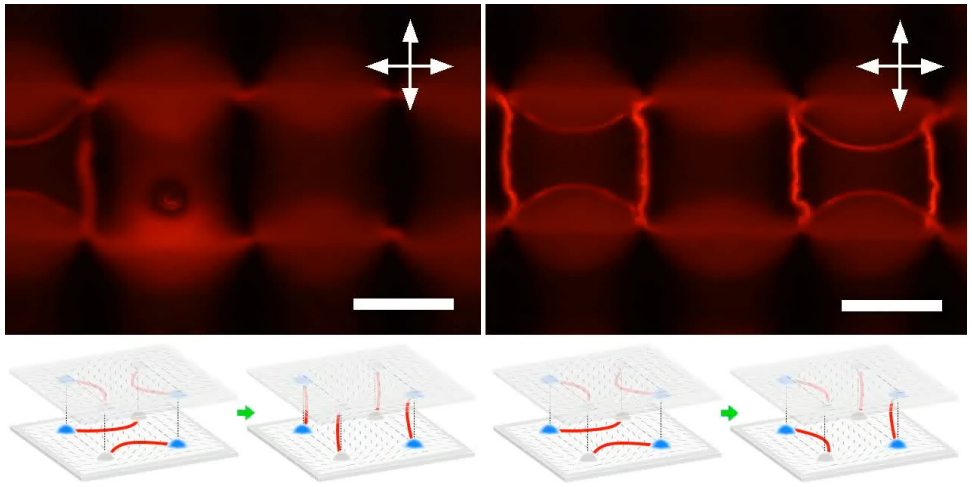
→Topological order can be vividly seen, engineered & exploited, e.g. in unconventional computing!!!

C. Meng, J.-S. Wu, Ž. Kos, J. Dunkel, C. Nisoli, and I. I. Smalyukh. *Phys. Rev. X* 15, 021084 (2025)

Quazi-3D vortex lattices for “bilayer” pinning sites



→**Transitions between topological states?**



→Disclinations/vortices can be along surface normal or in-plane

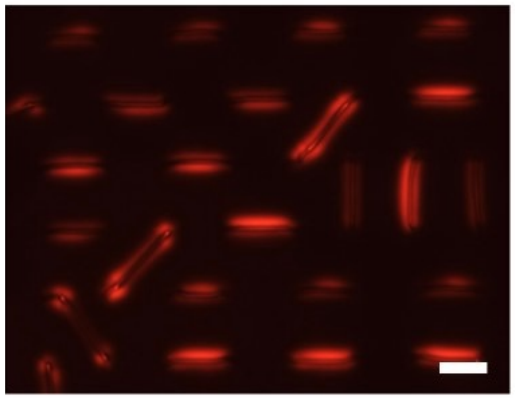
→Laser tweezers mediate “topological surgeries”

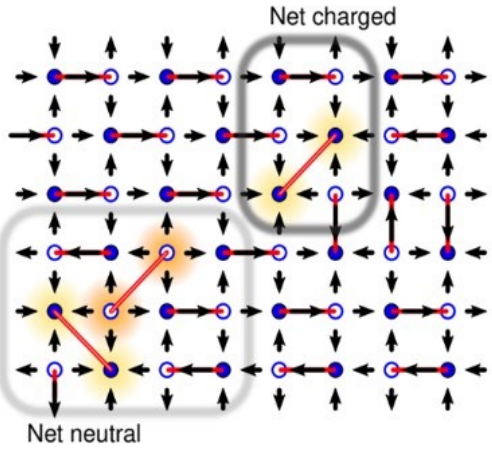
→Large space of states can be explored

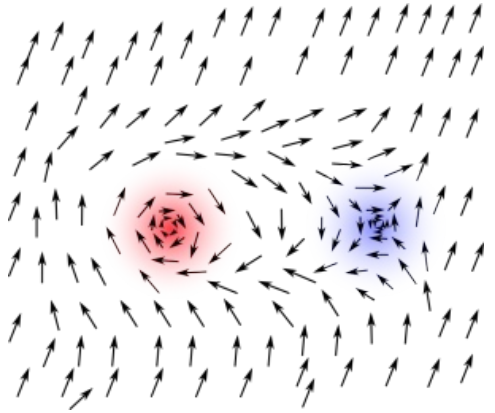
C. Meng, J. Wu, Z. Kos, J. Dunkel, C. Nisoli & I.I. Smalyukh, *Phys. Rev. X* **15**, 021084 (2025)

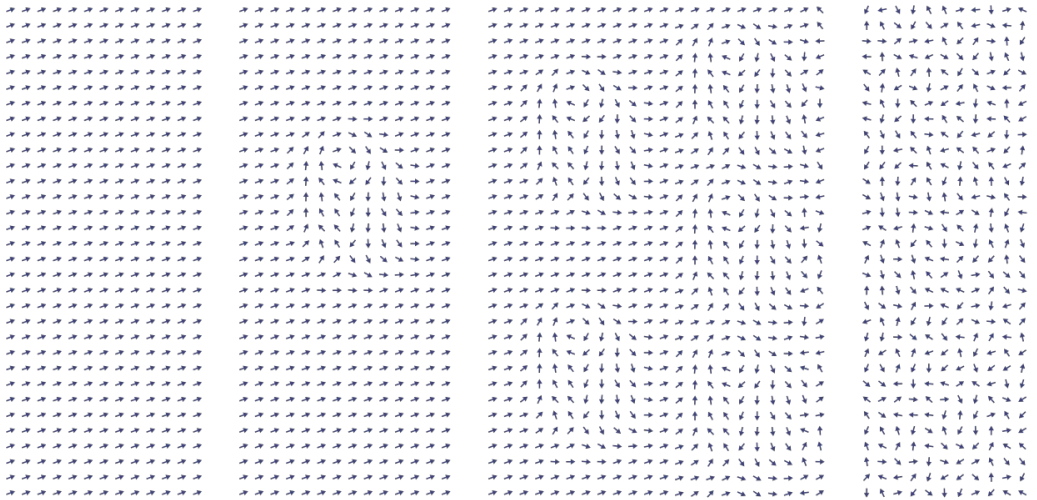
Topological defect mediated tweezers-guided phase transitions

→Kosterlitz & Thouless Nobel Prize
→Un-binding of defects by thermal energy
→A different type of phase transition between topological order states!
→Topological order & phase transitions in combinatorial “dimer model” lattices of of defects



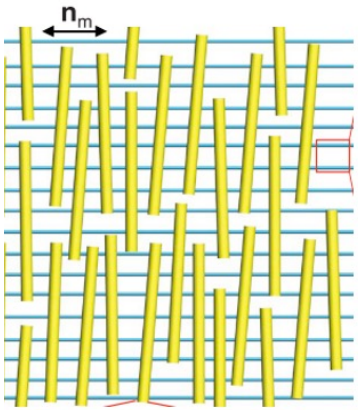


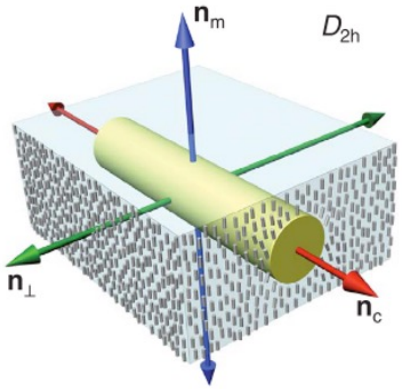


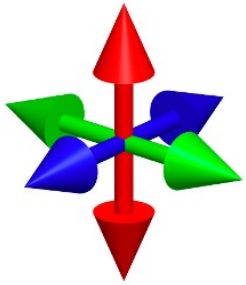


https://abeekman.nl/wp-uploads//vortex_unbinding.png

Low-symmetry order parameter spaces

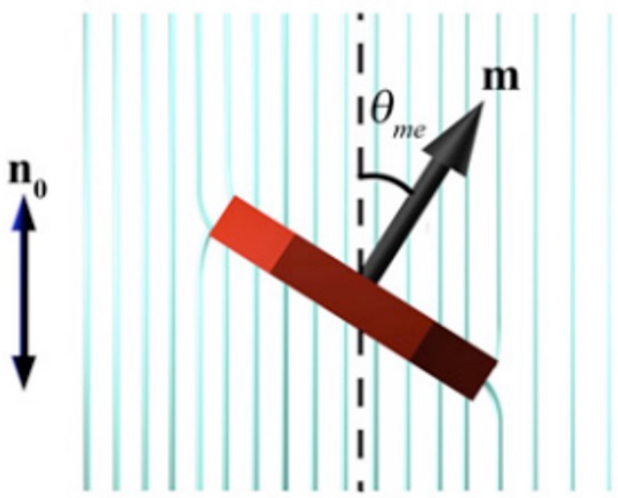


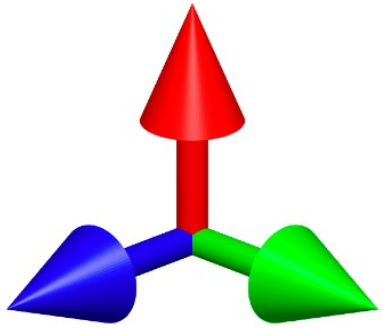




$SO(3)/D_2=\mathbb{S}^3/Q$

$\pi_1(SO(3)/D_2) = Q_8 = \{\pm 1, \pm\lambda, \pm\tau, \pm\chi\}$

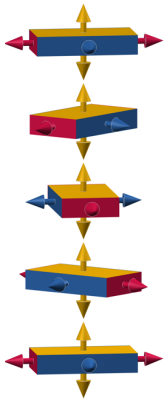


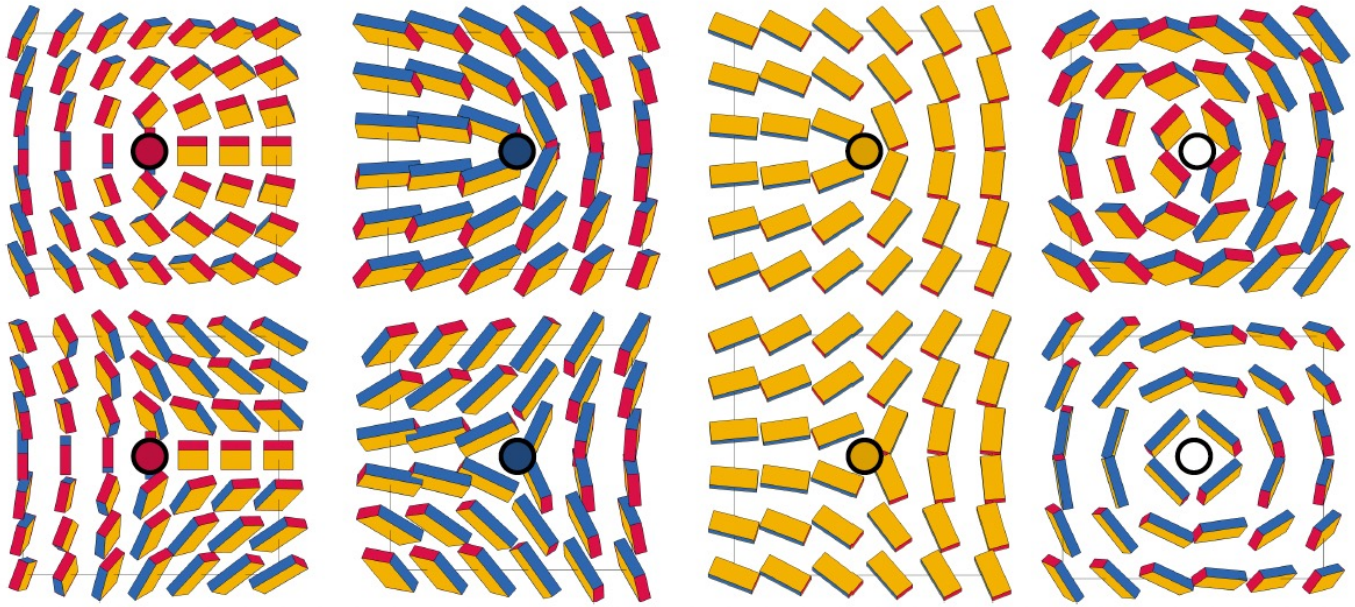


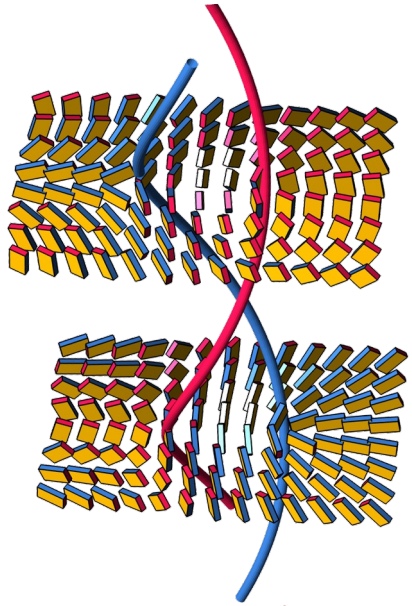
$SO(3)=\mathbb{S}^3/\mathbb{Z}_2$

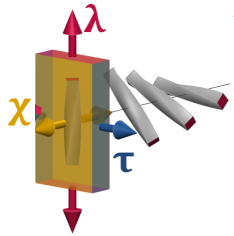
Nonabelian vortices: braids, links, knots & arrays

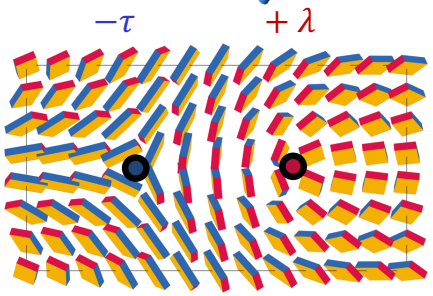
$\pi_1(SO(3)/D_2) = Q_8 = \{\pm 1, \pm\lambda, \pm\tau, \pm\chi\}$







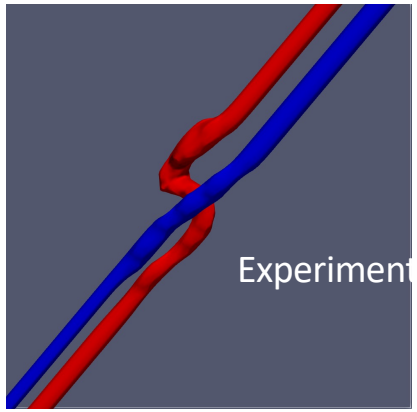


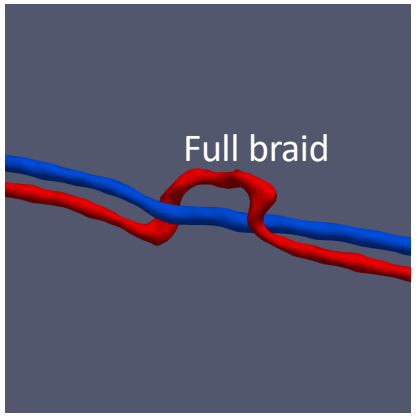


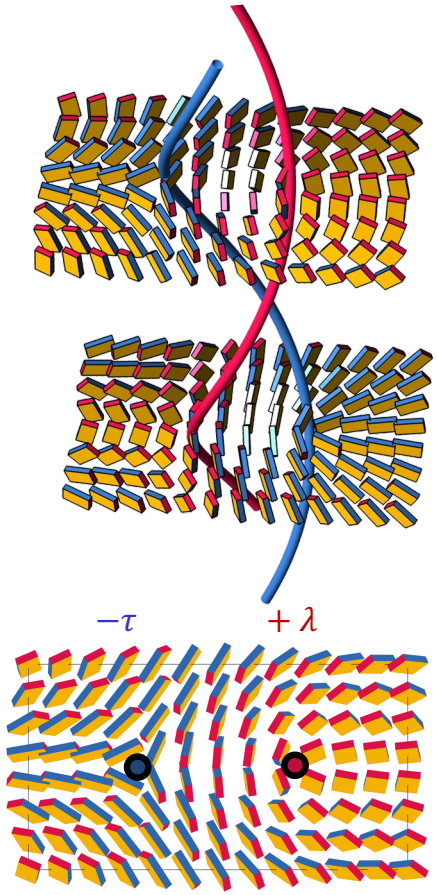
Wu, Valenzuela, Bowick, & Smalyukh. *Physical Review X* **15**, 021036 (2025).

Nonabelian vortices: braids, links, knots & arrays

$\pi_1(SO(3)/D_2) = Q_8 = \{\pm 1, \pm\lambda, \pm\tau, \pm\chi\}$

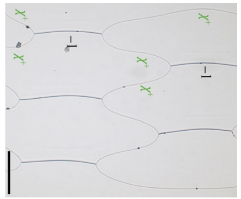


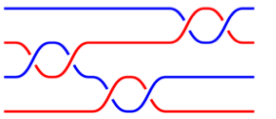


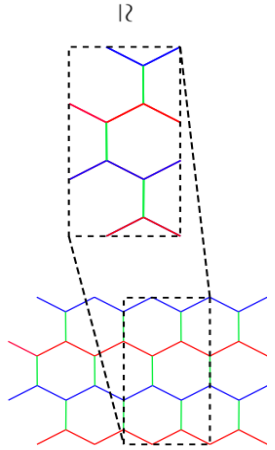


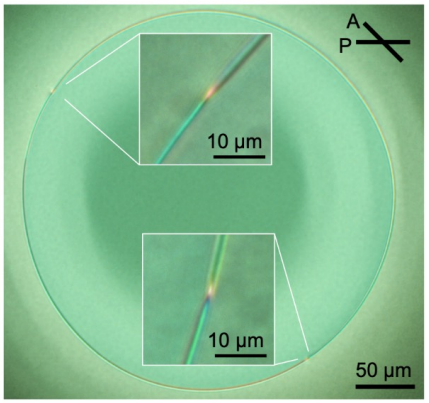
Junctions follow multiplication nonabelian defect algebra

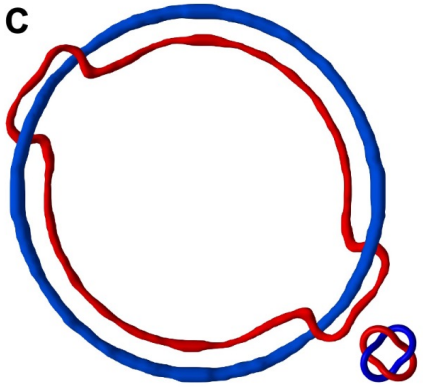
$\times$	1	-1	λ	$-\lambda$	τ	$-\tau$	χ	$-\chi$
1	1	-1	λ	$-\lambda$	τ	$-\tau$	χ	$-\chi$
-1	-1	1	$-\lambda$	λ	$-\tau$	τ	$-\chi$	χ
λ	λ	$-\lambda$	-1	1	χ	$-\chi$	$-\tau$	τ
$-\lambda$	$-\lambda$	λ	1	-1	$-\chi$	χ	τ	$-\tau$
τ	τ	$-\tau$	$-\chi$	χ	-1	1	λ	$-\lambda$
$-\tau$	$-\tau$	τ	χ	$-\chi$	1	-1	$-\lambda$	λ
χ	χ	$-\chi$	τ	$-\tau$	$-\lambda$	λ	-1	1
$-\chi$	$-\chi$	χ	$-\tau$	τ	λ	$-\lambda$	1	-1









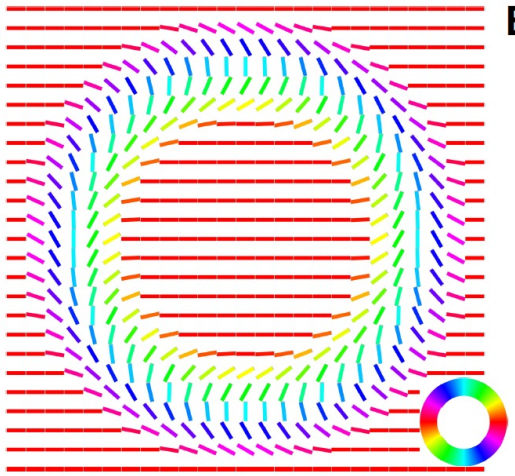


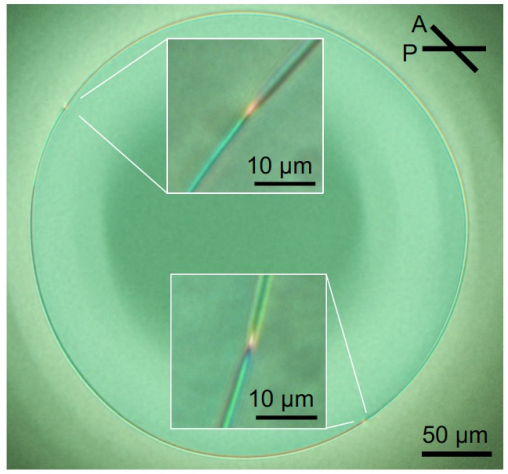
→Particle-like nonabelian vortex knots

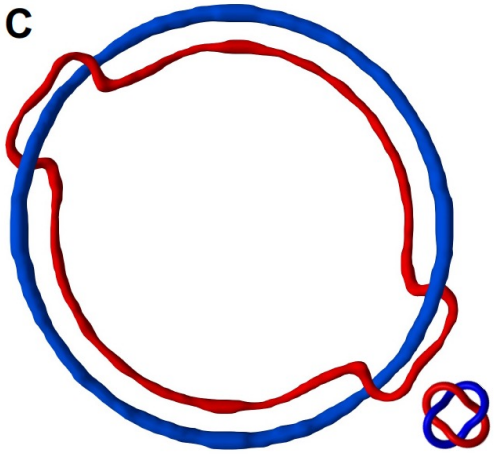
Wu, Valenzuela, Bowick, & Smalyukh. *Physical Review X* **15**, 021036 (2025).

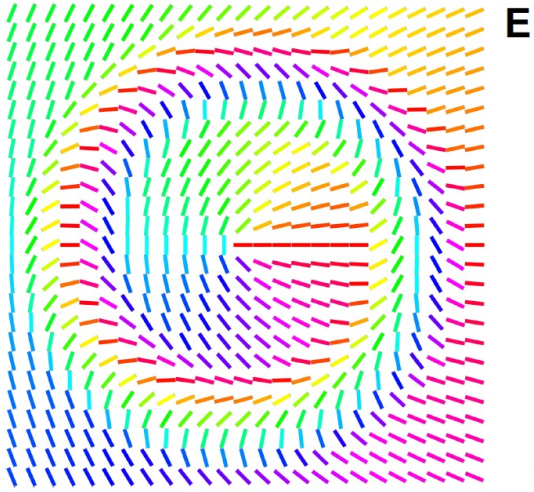
Wu, Valenzuela, Chen, Bowick, & Smalyukh (2026).

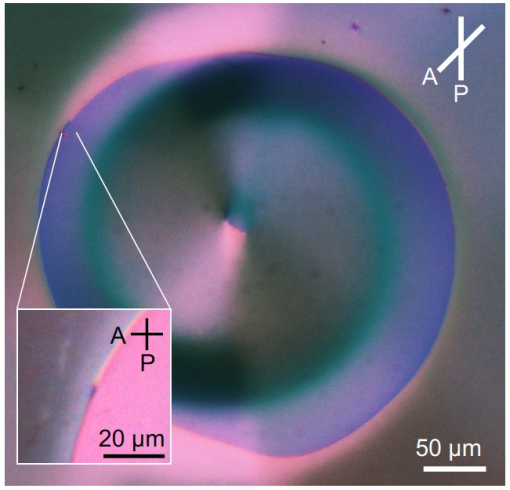
Closing loops of nonabelian defect lines

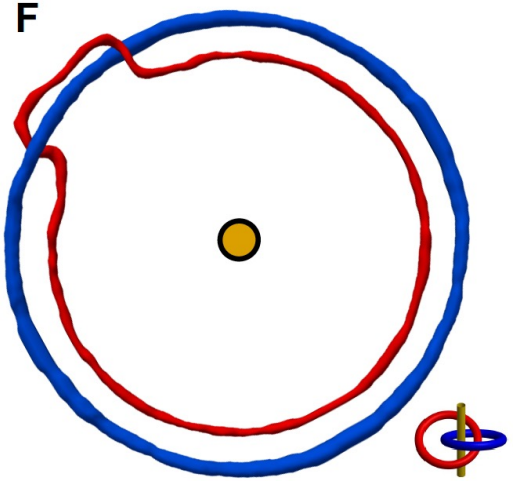


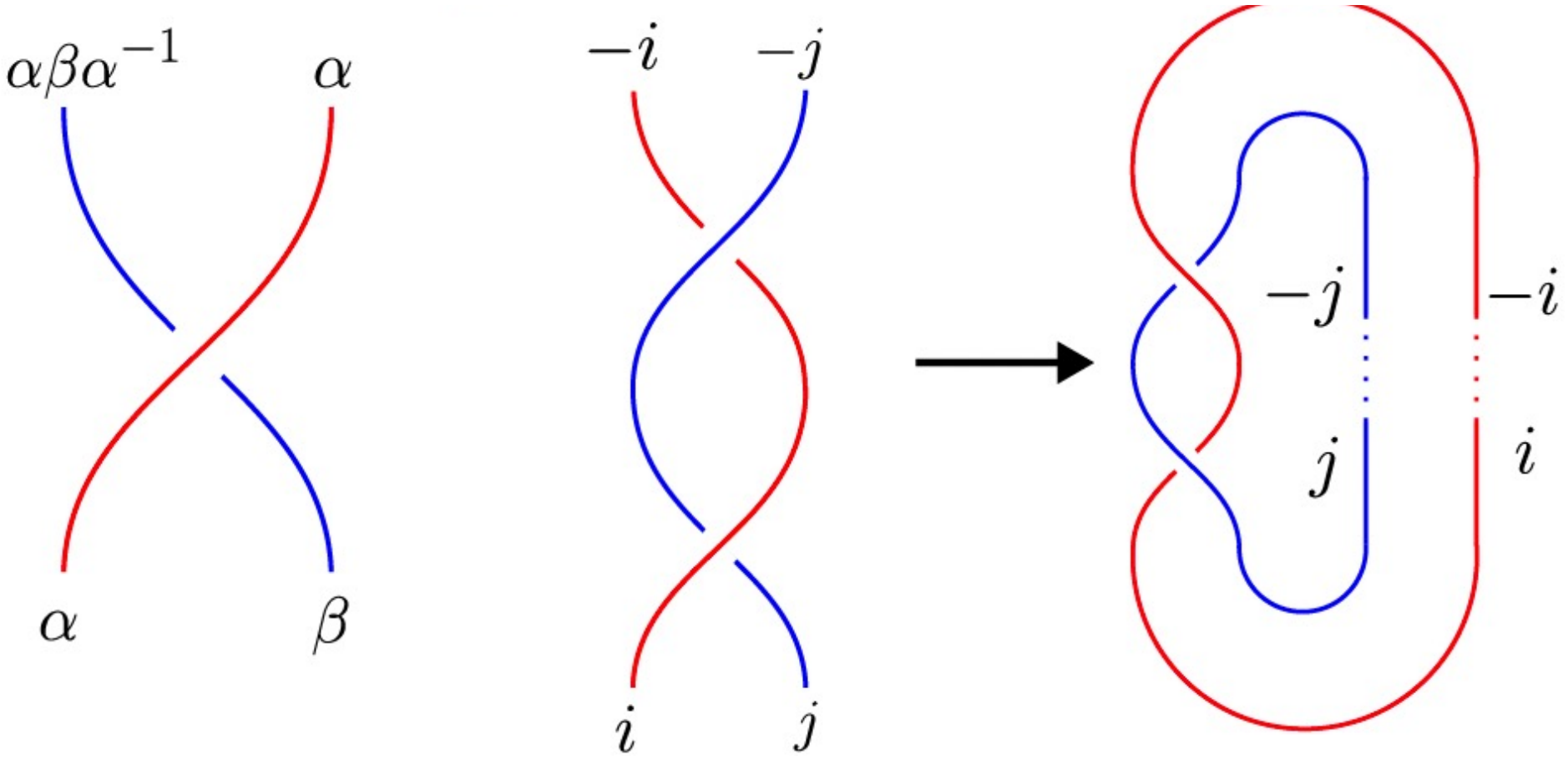






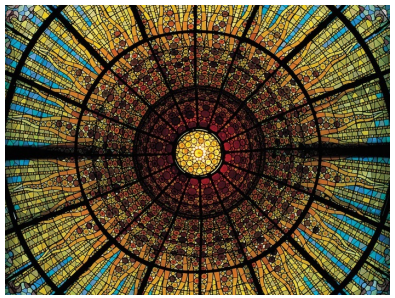





$\alpha\beta\alpha^{-1}$
α
α
β
$-i$
$-j$
i
j
$-j$
$-i$
j
i

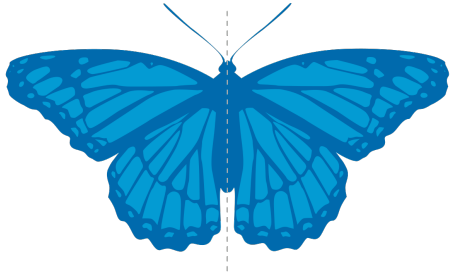
What is space symmetry?

An object that is invariant under some transformations



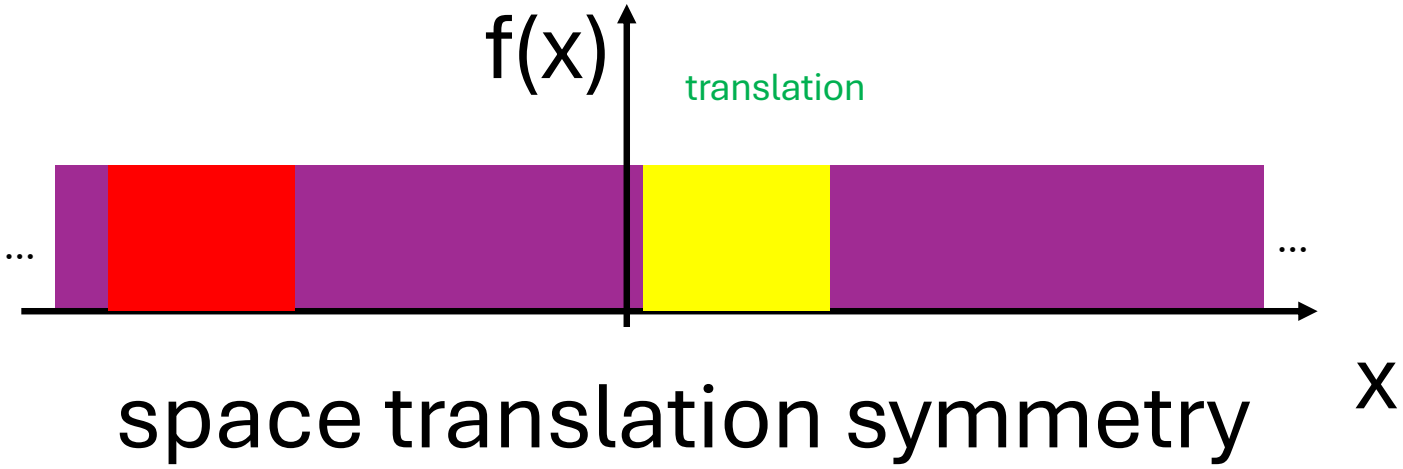
rotation

rotation symmetry



reflection

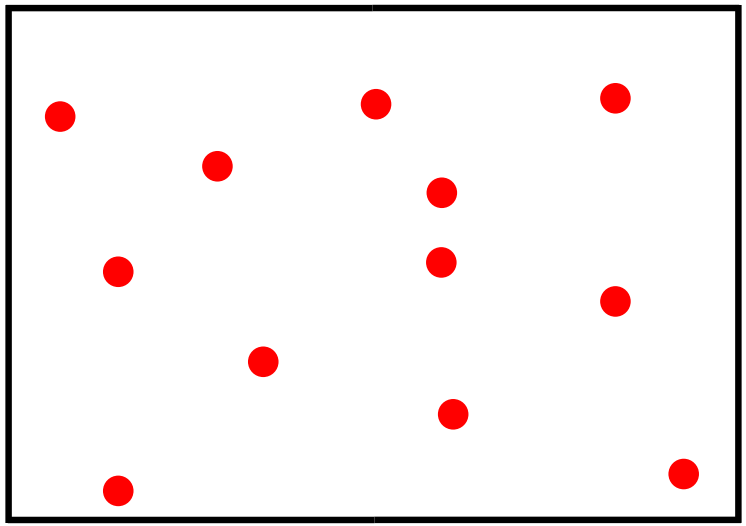
mirror symmetry

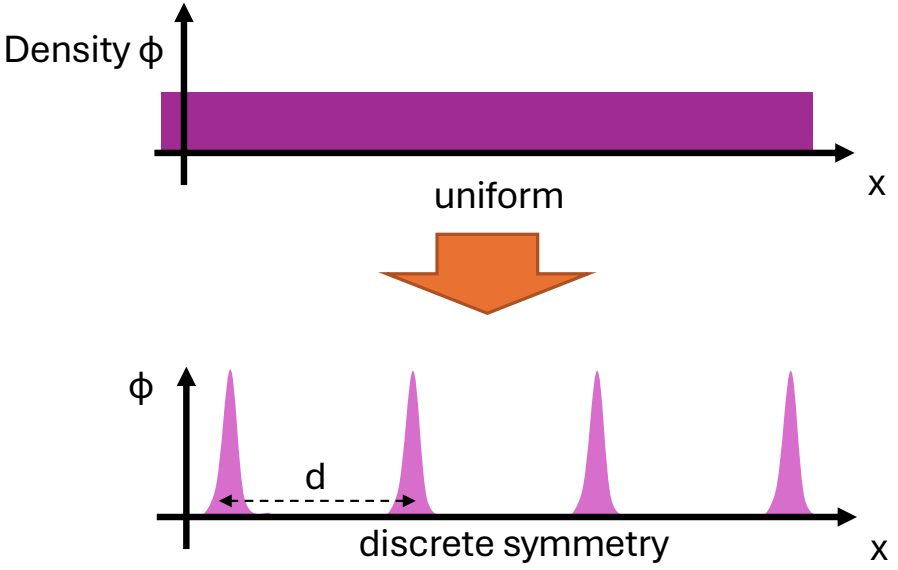


space translation symmetry

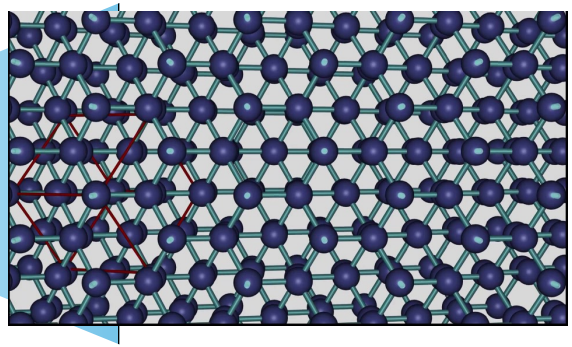
What is symmetry breaking ?

Spontaneous symmetry breaking





crystals



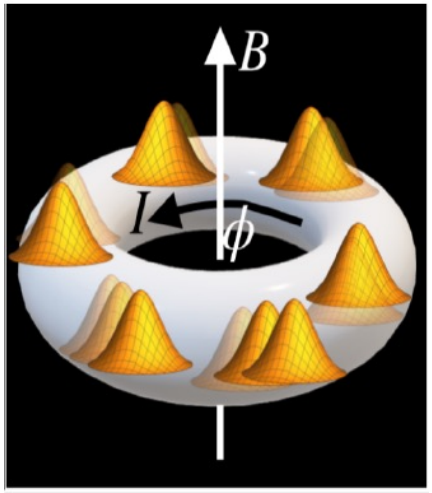
⇨ **Discrete space translation symmetry**

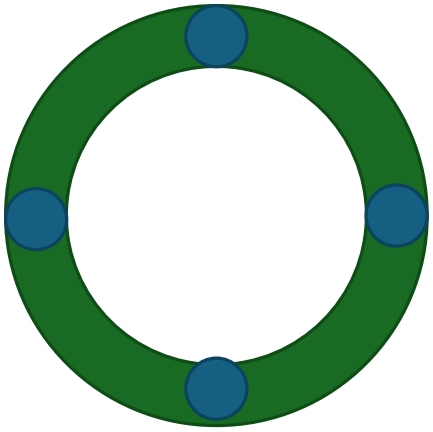
and

⇨ **symmetry breaking**

Time Crystals

Frank Wilczek

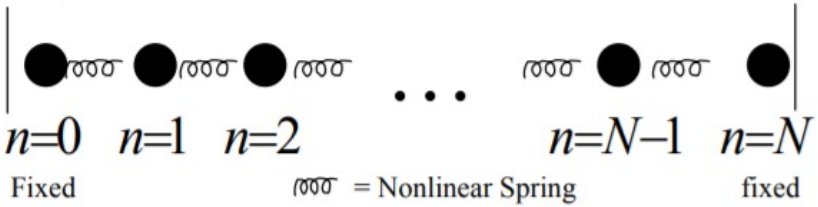




T=2πR/v

By applying a perpendicular magnetic field, these solitons will move at a constant speed

FPUT model:



$$V(x) = \tfrac{1}{2} kx^2 + \alpha/3\, x^3 + \beta/4\, x^4$$

$$H \sim P^2 + P^3 + P^4$$

$$P = \frac{\partial L}{\partial \theta}$$

PhysRevLett.109.160401,
PhysRevLett.109.160402

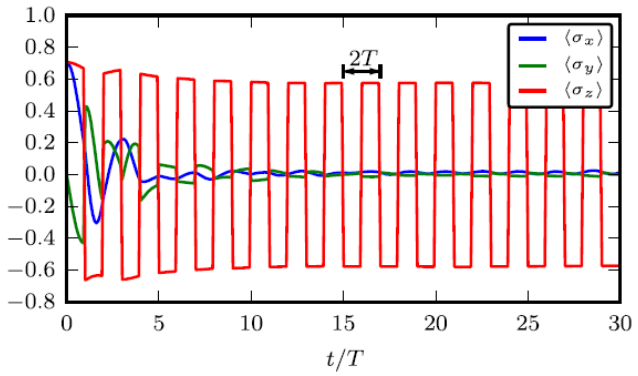
Discrete (Floquet) time crystals

⇨ **Definition:**

$$H(t)=H(t+T)$$

$$t \rightarrow t + nT$$

n is an integer larger than 1

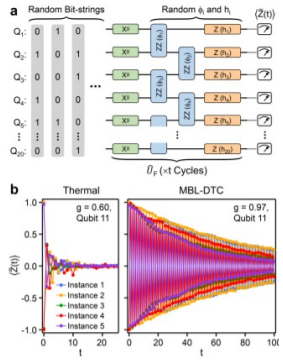


Physical review letters 117.9 (2016): 090402.

Basic rules:

- ⇨ **1 few body**
- ⇨ **2 stable**
- ⇨ **3 don't not exchange energy and entropy/ introduce dissipation**

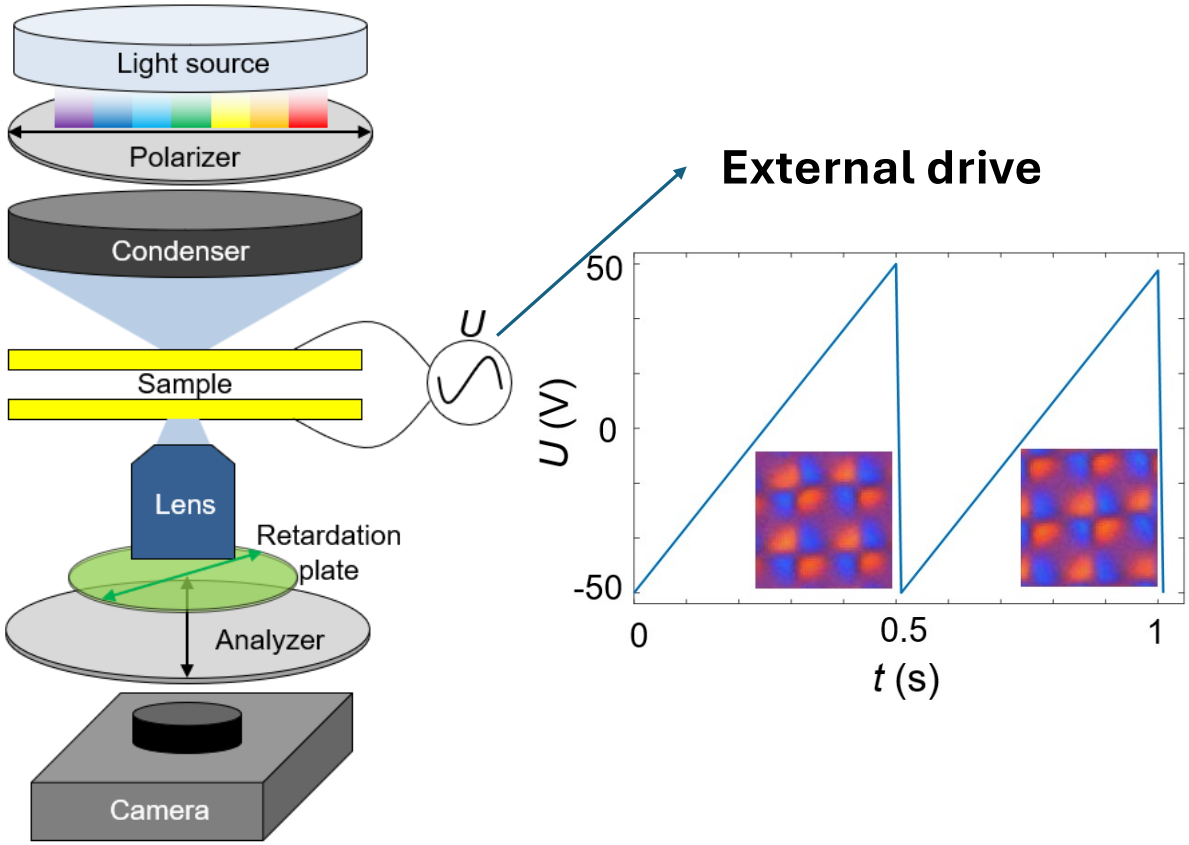
Nature 543.7644 (2017): 217-220.
Science 374.6574 (2021): 1474-1478.



Science 372.6547 (2021): 1192-1196.
Nature 601.7894 (2022): 531-536.

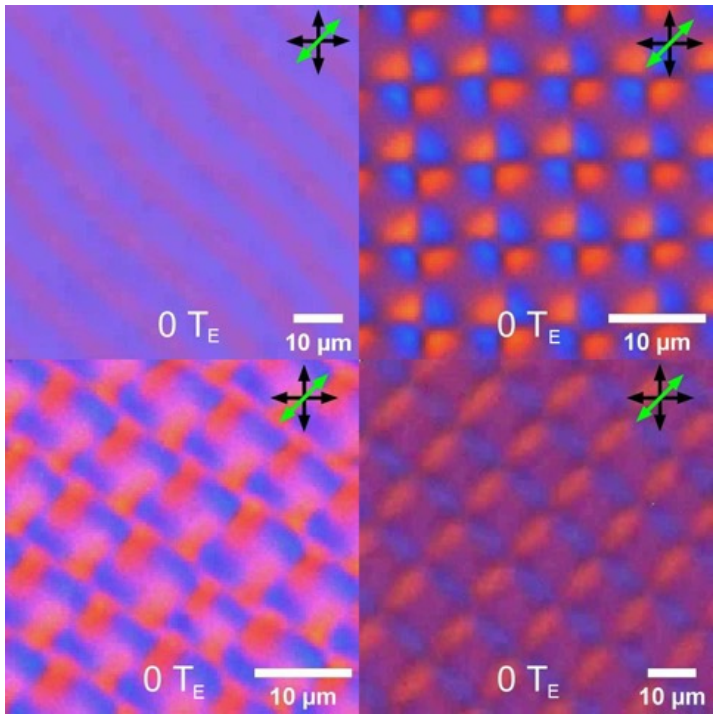
Classical discrete time crystals in a liquid crystal system

Experiment setup

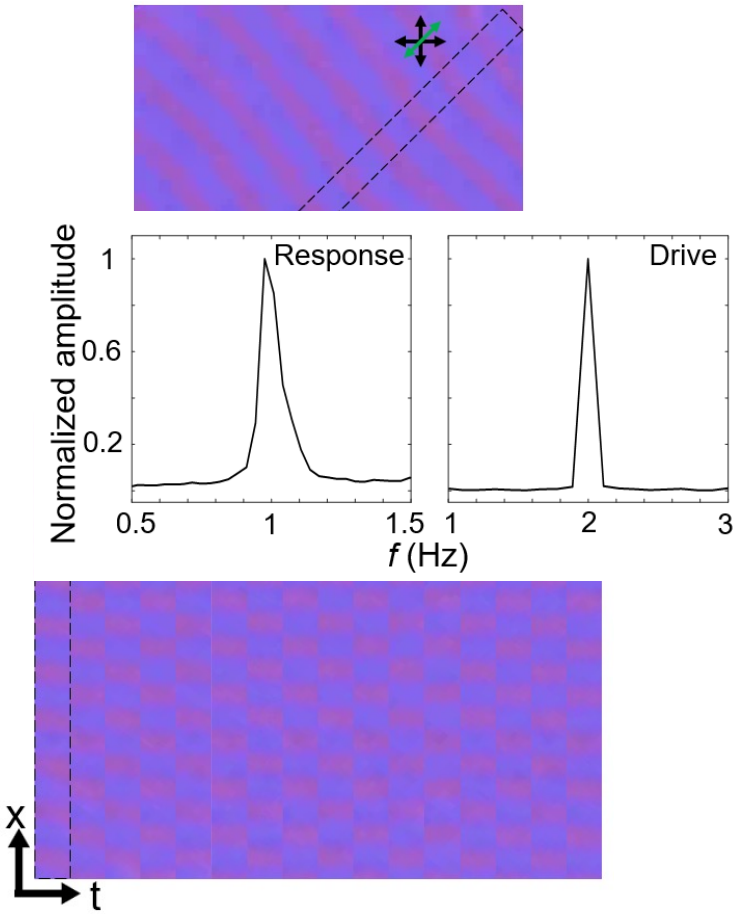


External drive

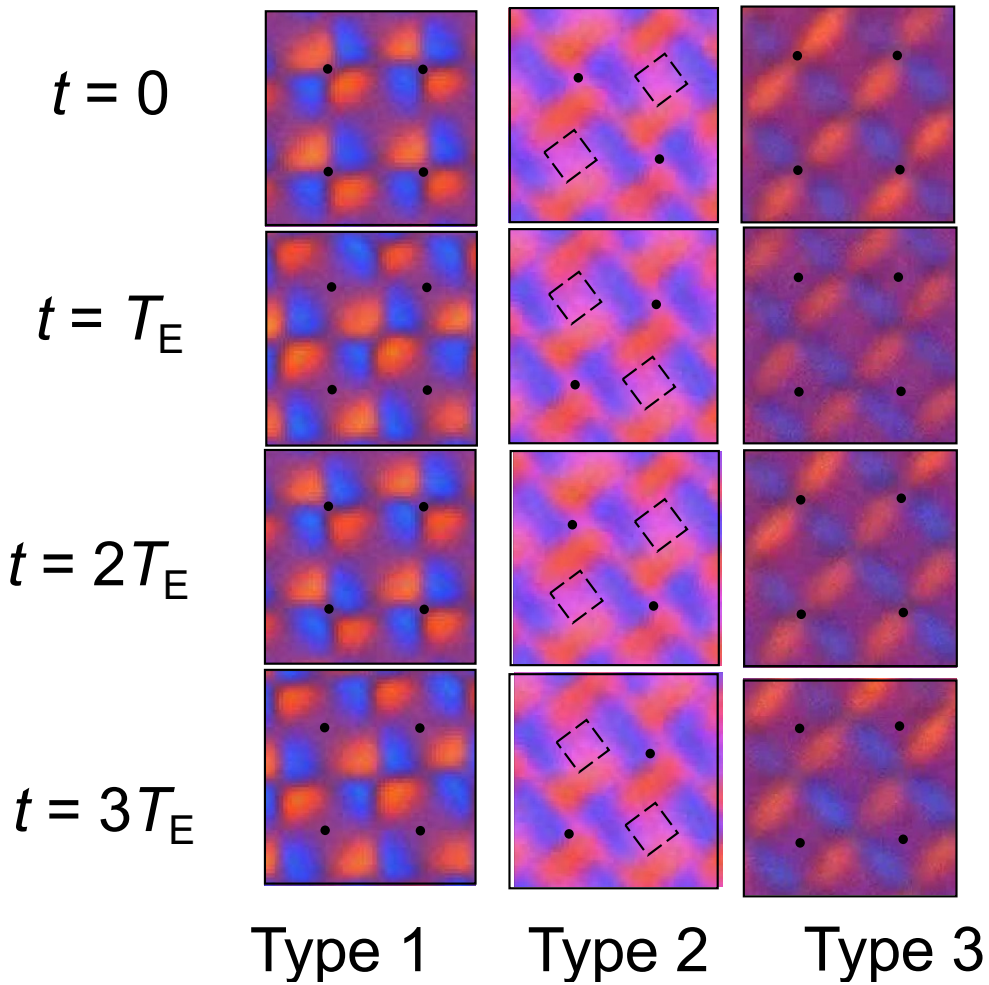
Results



1+1D DSTC



2+1D DSTC



Mechanism of 1+1D DSTC

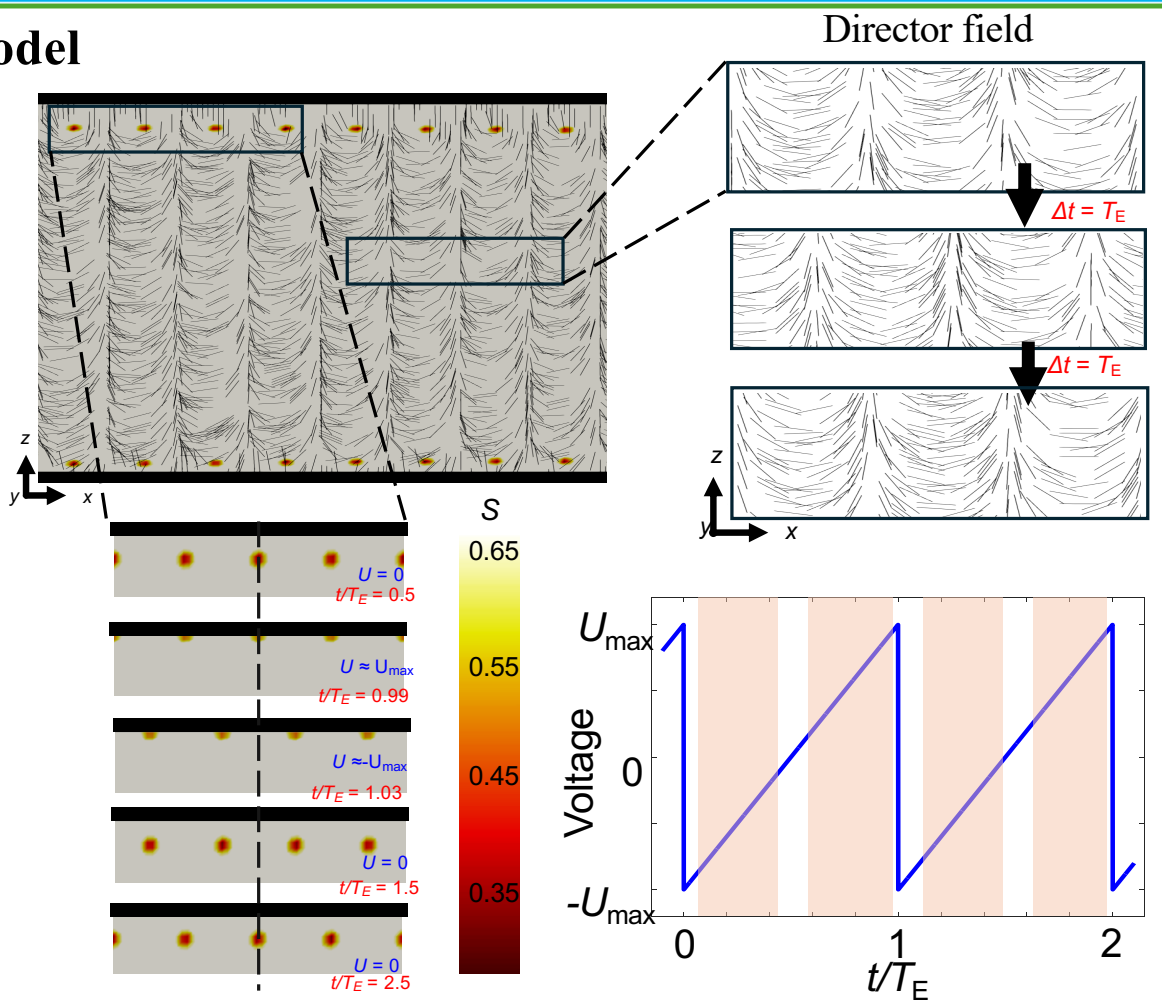
Ladau-de Gennes free energy model

Elastic term

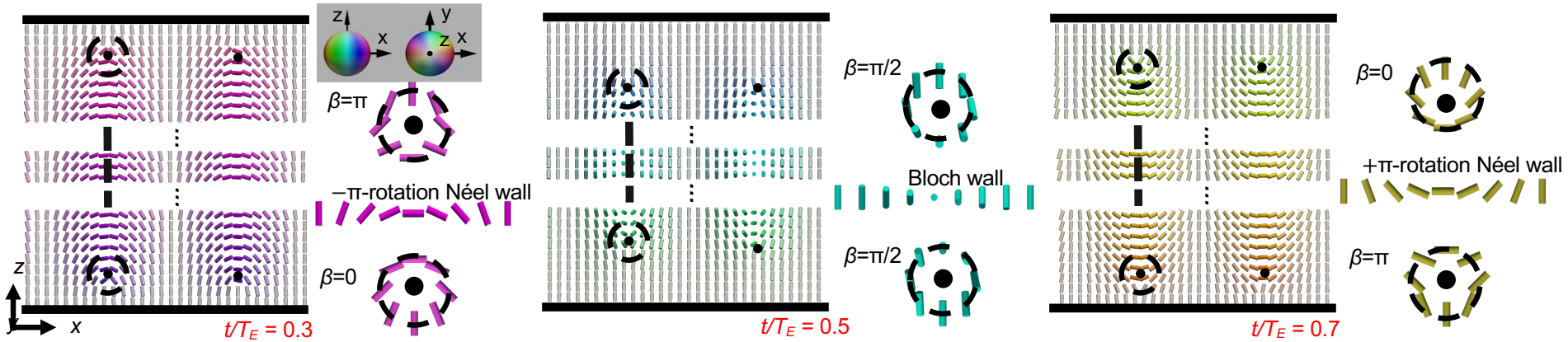
$$F_{\text{elastic}}^{\text{LdG}} = \int \mathrm{d}^3\mathbf{r}\left\{\frac{L_1}{2}\frac{\partial Q_{ij}}{\partial x_k}\frac{\partial Q_{ij}}{\partial x_k} + \frac{L_2}{2}\frac{\partial Q_{ij}}{\partial x_j}\frac{\partial Q_{ik}}{\partial x_k} + \frac{L_3}{2}\frac{\partial Q_{ij}}{\partial x_k}\frac{\partial Q_{ik}}{\partial x_j} + \frac{L_6}{2}Q_{ij}\frac{\partial Q_{kl}}{\partial x_i}\frac{\partial Q_{kl}}{\partial x_j} + \frac{4\pi}{p}L_4\varepsilon_{ikl}Q_{ij}\frac{\partial Q_{lj}}{\partial x_k}\right\}$$

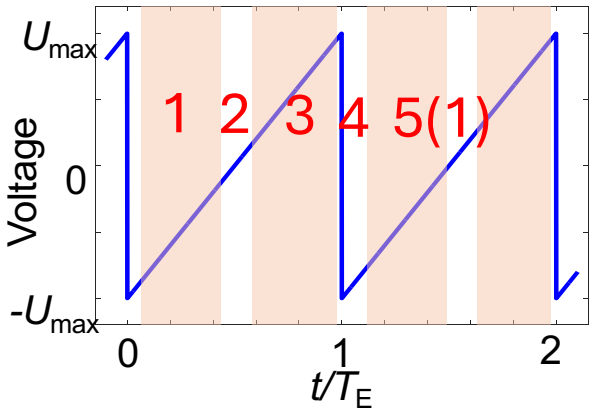
Electric term

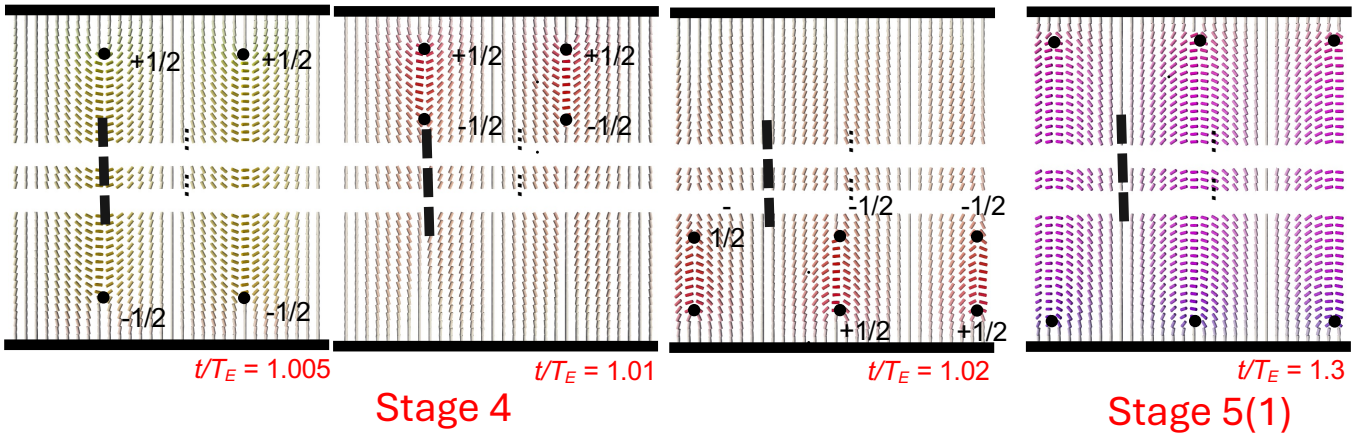
$$F_{\text{electric}}^{\text{LdG}} = \int \mathrm{d}^3\mathbf{r}\left\{-\frac{1}{2}\varepsilon_0\bar{\varepsilon}E_i^2 - \frac{1}{3}\varepsilon_0\varepsilon_{\text{a}}^{\text{mol}}Q_{ij}E_iE_j + \zeta_1\frac{\partial Q_{ij}}{\partial x_j}E_i + \zeta_2 Q_{ij}\frac{\partial Q_{jk}}{\partial x_k}E_i\right\}$$



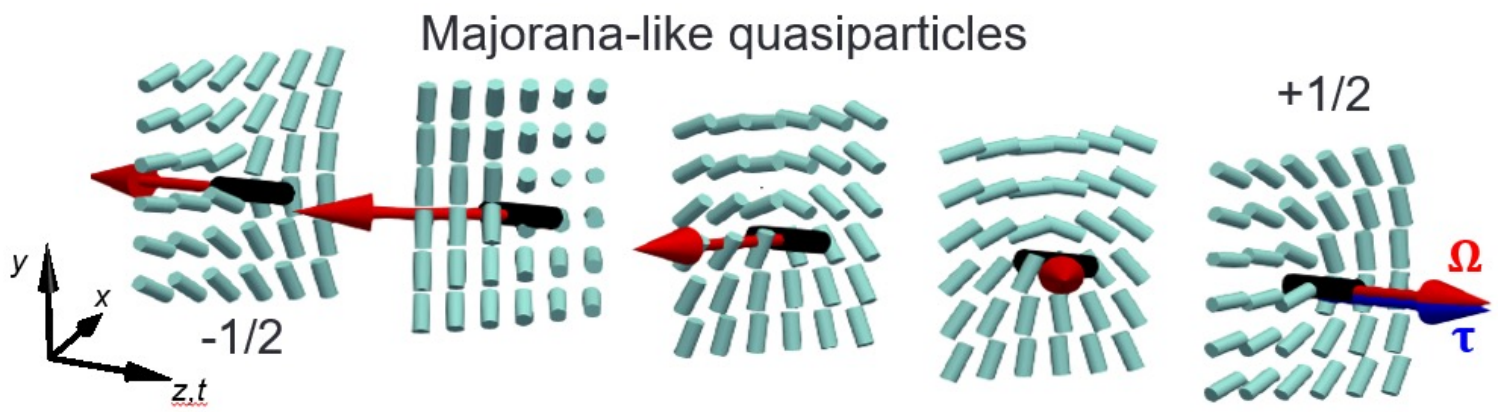
Period doubling mechanism







Majorana-like quasiparticles



Majorana Operator $D = ie_2\frac{\partial}{\partial t} + ie_2e_1\frac{\partial}{\partial x} - ie_1m.$

Solving $D\psi = 0,$

For a rest solution (momentum equals 0)

$$\psi = m\begin{pmatrix} -\sin(\theta) & \cos(\theta) \\ \cos(\theta) & \sin(\theta) \end{pmatrix}$$

where $\theta = -Et = -mt.$

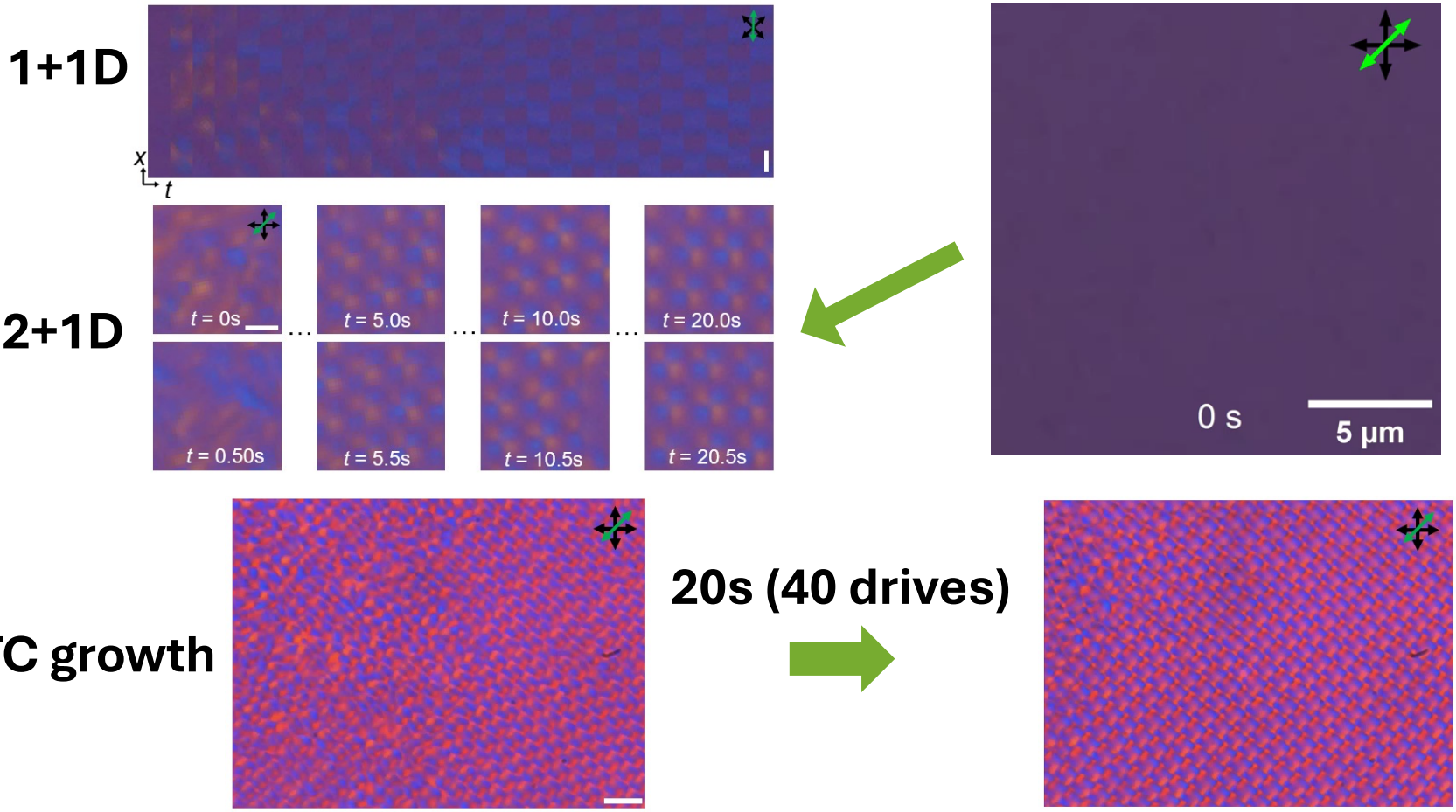
In LCs, a half integer disclination can be defined:

$$\mathbf{n}=\begin{pmatrix} \cos(\beta/2) & \sin(\beta/2) \\ -sin(\beta/2) & \cos(\beta/2) \end{pmatrix}$$

twist angle $\beta=\cos^{-1}(\boldsymbol{\tau}\cdot\boldsymbol{\Omega})$,

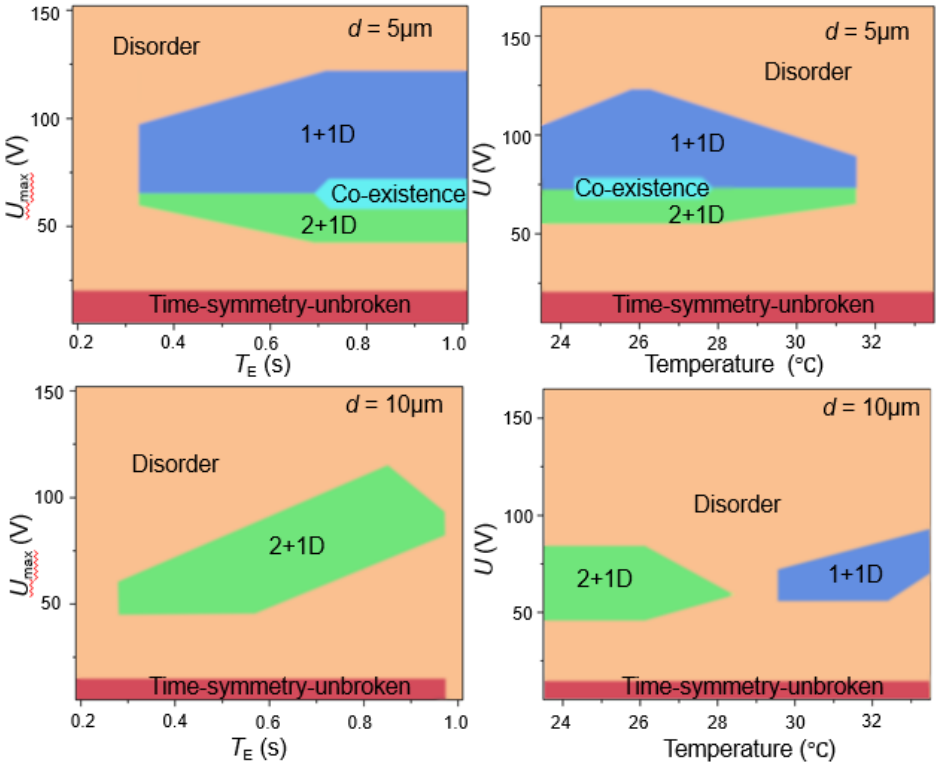
Majorana quasiparticles and topological phases in 3D active nematics.
Head, L. C. et al. *Proc. Natl. Acad. Sci.* **121**, e2405304121 (2024).
Three-dimensional active defect loops.
Binysh, J., Kos, Ž., Čopar, S., Ravnik, M. & Alexander, G. P. Phys. Rev. Lett. 124, 088001 (2020).

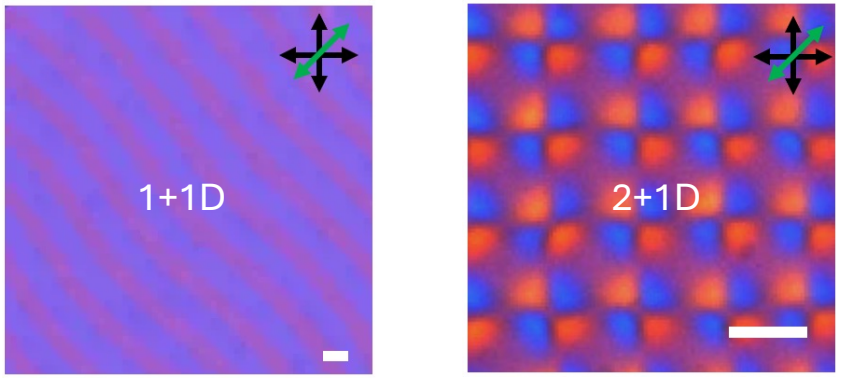
Spontaneously emergence of DSTC

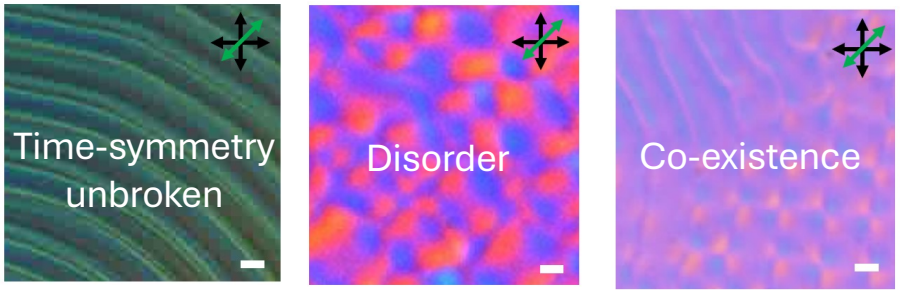


Different phases of DSTC

⇨By changing voltage, external period and temperature

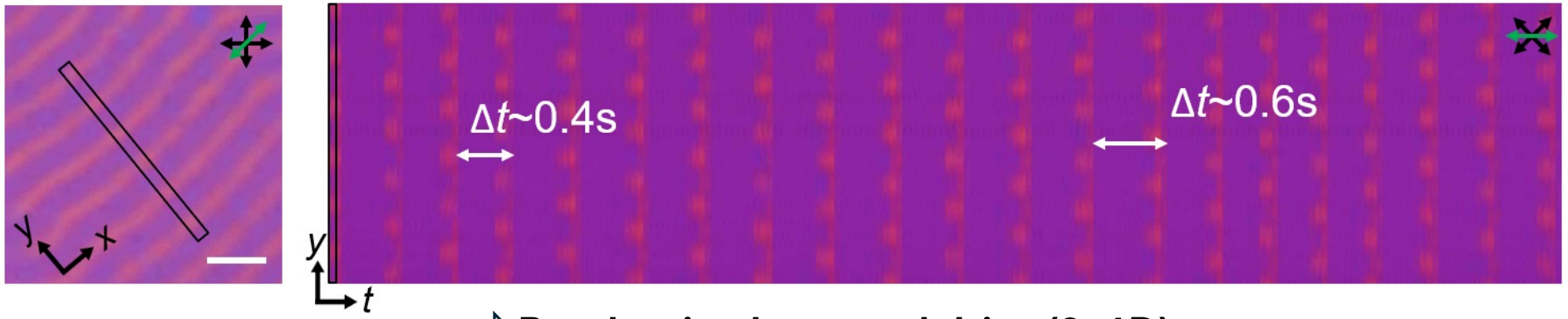






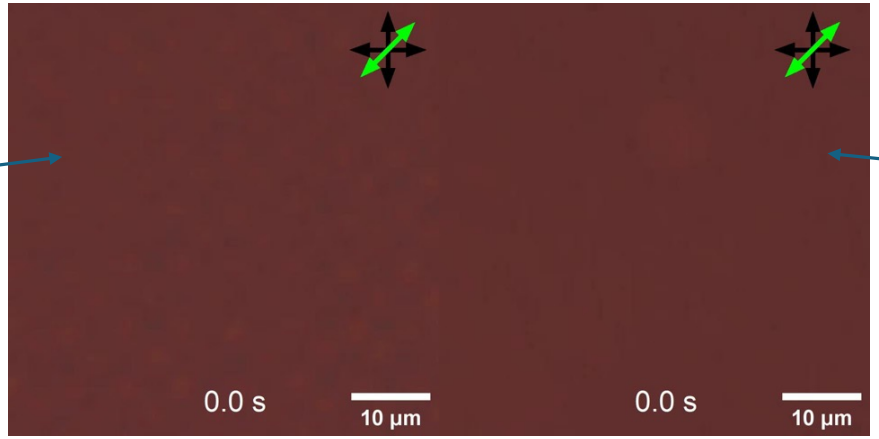
Robustness (Rigidity) of DSTC

⇨ **Randomized external drive**



⇨ **Randomized external drive (2+1D)**

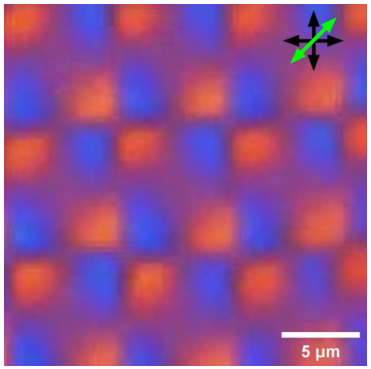
ΔT_E randomly distribute within $[-0.2\bar{T}_E, 0.2\bar{T}_E]$

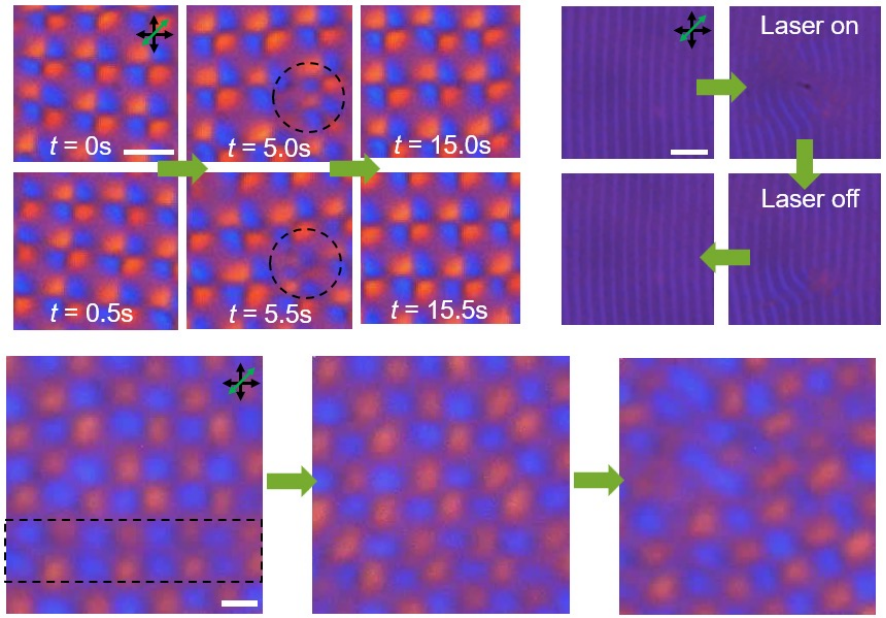


ΔT_E randomly distribute within $[-0.4\bar{T}_E, 0.4\bar{T}_E]$

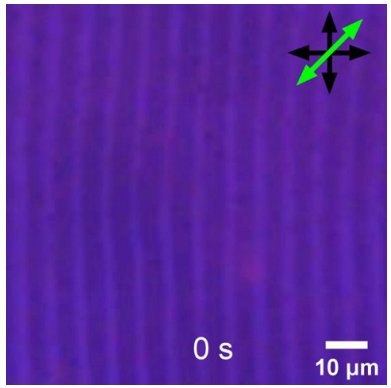
Defect (impurity) region of DSTC

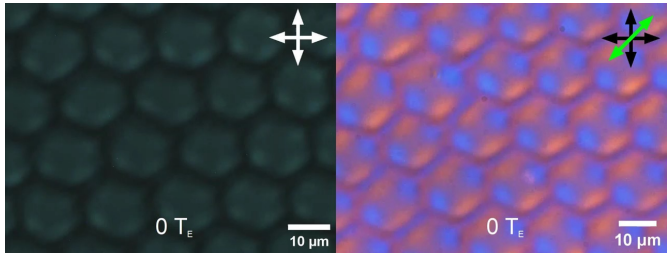
⇨ Emergence of a defect region





⇨ Optical laser tweezer induced defect





⇨ Quasi-hexagonal lattice (fractional space time crystal)

Conclusions

→LC topological defects & solitons as quasiparticles

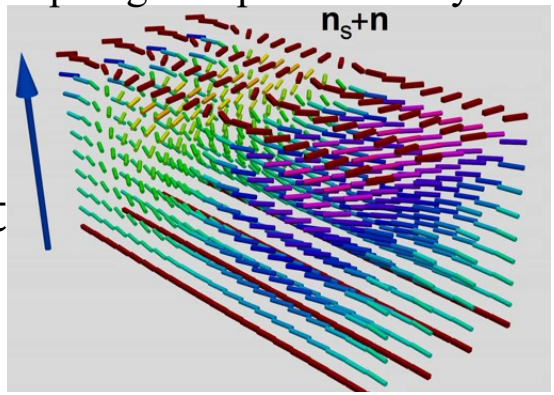
→Crystals of disclinations & knot solitons

→Emergent topological order in combinatorial vortex latt

→Electro-fusion, -fission & emergent dynamics of knots

→Entangled states & topological order in 3D

→Topological space-time crystals



Zhao & Smalyukh. *Nat. Mater.* **24**, 1802 (2025)

Thank you !!!