Brussels, 17 May 2024

COST 047/24

**DECISION**

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Subject: Memorandum of Understanding for the implementation of the COST Action “Topological textures in condensed matter” (Polytopo) CA23134

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The COST Member Countries will find attached the Memorandum of Understanding for the COST Action Topological textures in condensed matter approved by the Committee of Senior Officials through written procedure on 17 May 2024.

**MEMORANDUM OF UNDERSTANDING**

For the implementation of a COST Action designated as

**COST Action CA23134**
**TOPOLOGICAL TEXTURES IN CONDENSED MATTER (Polytopo)**

The COST Members through the present Memorandum of Understanding (MoU) wish to undertake joint activities of mutual interest and declare their common intention to participate in the COST Action, referred to above and described in the Technical Annex of this MoU.

The Action will be carried out in accordance with the set of COST Implementation Rules approved by the Committee of Senior Officials (CSO), or any document amending or replacing them.

The main aim and objective of the Action is to address research questions related to topological textures from a broad perspective across condensed matter and related systems. The Action aims to develop a unified framework for topological textures, including textures in three dimensions. It will investigate and describe their statics and dynamics, their collective behaviour, and support their applications potential. This will be achieved through the specific objectives detailed in the Technical Annex.

The present MoU enters into force on the date of the approval of the COST Action by the CSO.

## OVERVIEW

**Summary**
The mathematical concept of topology is now widely applied in condensed matter systems providing a fresh perspective for the understanding and prediction of many physical phenomena. The Action focuses on techniques for the creation and annihilation of topological textures and on their static and dynamic properties that form a vibrant field of research.

A variety of topological textures are observed and are ubiquitous in the sciences (physics, materials, biology, chemistry). Examples are domain walls, vortices, skyrmions, and hopfions in bulk materials, with a big challenge now posed by the complexity added by the third dimension. Their dynamics and response to external probes are often counter-intuitive and they are linked to topology in surprising ways. These properties make them promising for applications in sensors, logic and non-Boolean devices, logic-in-memory computing, and neuromorphic or quantum computing applications. Related physical systems include magnets, ferroelectrics, optics and topological light, ultra-cold atoms and superfluids, superconductors, liquid crystal and similar systems, quantum systems with a topological band structure, and others.

The area of condensed matter benefits greatly from the interaction between experiments, theory, and computations on topological textures. Improved techniques allow for observations at the nanoscopic and picosecond scales. However, progress is driven by small and fragmented communities of physical scientists and by a separate community of applied mathematicians. This network will bring together in a wider network the separate sub-communities working on the various physical systems, it will combine expertise and boost synergies. It aims to develop a unified framework for topological condensed matter systems.

| **Areas of Expertise Relevant for the Action** | **Keywords** |
|---|---|
| ● Physical Sciences: Nanophysics: nanoelectronics, nanophotonics, nanomagnetism or classify<br>● Physical Sciences: Non-linear physics<br>● Physical Sciences: Soft condensed matter (e.g. liquid crystals)<br>● Mathematics: Theoretical aspects of partial differential equations | ● topological solitons<br>● skyrmions<br>● hopfions<br>● topological defects |

**Specific Objectives**
To achieve the main objective described in this MoU, the following specific objectives shall be accomplished:

Research Coordination
● Coordinate research across physical systems and create a synergetic forum and platform for researchers coming from different relevant physical systems. The Action aims to develop a common understanding of the subject matter across physics and mathematics.
● Create an active link between the experimental/applied community and theoretical research. The Action aims for two-way flow of information during the networking activities, for collaborations and corresponding publications between experimental/applied and theoretical researchers.

Capacity Building
● Build a community and achieve critical mass. Smaller communities on topological textures in various

physical systems will combine into a large interconnected and interdisciplinary community.
● Foster the development of a joint research agenda. The various successful research lines that are currently pursued by a number of smaller communities will be unified with the support of well-developed topology concepts. A joint research agenda for the field of topological textures in condensed matter will emerge.
● Train the next generation of researchers offering comprehensive knowledge and methodological tools. The Action aims for young physical scientists to gain a broad view of condensed matter and complement their intuition with a wide set of methodological tools. Also, young mathematicians will exploit their technical achievements through their broader interpretation.

# TECHNICAL ANNEX

## 1. S&T EXCELLENCE

### 1.1. SOUNDNESS OF THE CHALLENGE

#### 1.1.1. DESCRIPTION OF THE STATE OF THE ART

**Topological structures** of astonishing complexity appear in Nature at all scales, from hurricanes which cover hundreds of kilometers, to the meso-scales of optics or the biological realm, to nano-scale spin textures in magnetic materials. While their physical description, their dimensions, timescales and methods of investigation differ vastly, there are striking similarities in their mathematical description. Topology is a well-developed branch of mathematics classifying structures that are obtained from each other under continuous deformations such as bending, twisting or stretching. Topology concepts provide a universal language to describe phenomena across an expanding range of research areas in the physical sciences. Topological complexity is not merely a description but it can point to the very reason for the existence of novel physical phases, states, and phenomena.

The use of concepts from topology in physics has become widespread in the past few decades. The discovery and understanding of Quantum Hall Effects demonstrated that local symmetries are not sufficient to describe quantum phases of matter and long-range structures are needed. The Skyrme model provided nonlinear classical field equations that gave stable topological soliton solutions. Following these early examples, the interest in topological structures has been fueled by their application in quantum effects and by the observations of a plethora of topological textures in various physical systems. The progress was recognised, e.g., by the award of the 2016 Nobel prize in Physics for "Topological Phase Transitions and Topological Phases of Matter." This referred to both the classical topological phase transitions, as well as the Haldane model. The continuous interplay between advancements in classical and quantum systems with contributions from mathematical studies has been pivotal for the growth of this field.

Despite the similarity in mathematical concepts, there is still a lack of coordination and even awareness between different branches of the physical and mathematical sciences where topology plays a primary role. This network brings together researchers working on different physical systems and from many disciplines, with the aim to further develop this universal language and through cross-fertilization to contribute to the rapid development of versatile and powerful ideas within topological complexity theory and its experimental realisations and applications. The Action focus is on a shared platform between classical and quantum physics, and mathematics.

The Action will focus on **topological textures** that are commonly observed in a number of different physical systems. Superconductors and Bose-Einstein condensates (BEC) of atoms, exciton-polaritons, or other quasi-particles constitute systems where textures, such as **quantized vortices,** are observed at the mesoscopic scale [Co,La]. These are **topological solitons** in two space dimensions and they can be manipulated. For example, vortex-antivortex pairs behave as propagating solitons and may interact with the superflow in a quantifiable way [Kw]. Magnets have shown particularly rich behaviour with ubiquitous observations of topological solitons, such as **skyrmions** [Ba]. The perceived disadvantage of skyrmions related to their complexity [Sc] has turned into an advantage once they were found to nucleate spontaneously in chiral magnets. This gave inspiration for corresponding studies in ferroelectrics [HO] and in optics [Ts], where experimental and theoretical efforts are rapidly expanding. Especially, optical systems have the advantage of experimental simplicity and versatility [Oz]. Liquid crystals come in many forms and support a correspondingly wide range of structures or **defects** [Al,Bal] that have been observed and manipulated experimentally [HS]. Topological textures in momentum space play a crucial role in a large number of insulators, semimetals, and metals [Ka,Ha,NN], while band structures with geometrical and topological features are realised in cold atomic gases [CDS]. Superconductors with a topologically non-trivial band structure are predicted to host Majorana bound states [Na] in the vicinity of topological textures such as superconducting vortices [Fu] or magnetic skyrmions [Re].

The above outline defines the scientific breadth of the Action at its start. The field of topological physics is very broad and the Action will strive, during its lifetime, to include as broad a spectrum of systems as possible.

**Topological solitons** take many forms and a variety of textures can coexist in the same system or even in the same material. The inherent **nonlinearity** leads to physical and mathematical complexity and it also calls for sophisticated numerical analysis methods [Ab,Bao,Wa]. Furthermore, it makes it challenging to obtain a rigorous upscaling from microscopic to mesoscopic scales while keeping track of the topological structures. The complexity and the unusual dynamics related to topology have given

rise to proposals for applications in storage and transfer of information [Ba], logic gates [KD], quantum [No], and neuromorphic computing and implementation hardware for machine learning algorithms [So]. A great part of the work to date has focused on domain walls as described by models and theories in one space dimension, and on vortices, skyrmions and defects described by two-dimensional models. The Action will be based on expanding these studies and including textures in higher dimensions

In **quasi-two dimensional systems** (e.g., films), topological solitons can be represented as filaments. These may extend through the entire sample and they may have two open ends, as in skyrmion strings [Se] and dipoles [Sm], or one open end, as in chiral bobbers. In addition, the lines may interwind and they are then described by the mathematical theory of braids. The enrichment of experimental techniques in recent years allows to observe and map the complexity of such textures [Zh].

In bulk materials (in three space dimensions), topological soliton configurations, often called **Hopfions**, typically have a toroidal shape. They are represented as closed loops or rings, hence the term *vortex rings*. The most recognizable vortices in 3D are smoke rings in the air while we also have propagating rings in fluids and in superfluids [Jo]. Vortex rings have been observed in nonlinear optical media [Sg] and in trapped atomic BEC, e.g., in [Gi] where they are believed to play a crucial role in superfluid turbulence. Further examples are the Hopfions in magnetic materials [Do,Ke] and in ferroelectrics [Lu]. Analogous are the so-called biaxial torus pattern and the Saturn ring defect in liquid crystals [Al]. Topology in three space dimensions is quantified by the **Hopf index**, a topological invariant giving the linking number of any two contours where the order parameter takes a fixed value. These contours are the fibers in the fiber bundle theory underlying the Hopf index. In liquid crystals, states with a nonzero Hopf index were reported already in [Bo]. The Hopf fibration is further employed to explain the observed geometric phases obtained during the transformations of light polarization [Ci]. The complexity of Hopfions poses great challenges and makes their observation, theoretical description, and even numerical simulation very demanding.

Interactions between topological textures give rise to **collective behaviour** and pattern formation such as lattices of quantized vortices or magnetic skyrmions, clustering of co-rotating vortices in superfluids, dislocation patterning in crystals and others. The proliferation of vortices can signal topological phase transitions, dynamical transitions to turbulent flows or melting of order into disordered states. In driven-dissipative superfluids, the Kardar-Parisi-Zhang order competes with vortices giving rise to their unusual dynamical behaviour. Different collective states and dynamical regimes induced by topological textures have also been observed in **active matter**, which are systems of interacting anisotropic constituents forming various ordered states similar to liquid crystals, but which are intrinsically driven out of equilibrium by the self-propulsion of the constituents [SS]. Within the biological active matter spanning from cell tissues and organisms down to subcellular structures, such as mixtures of microtubules and protein motors, the prevailing topological textures are topological defects. They play a crucial role as centres for morphological events and reorganisation, sources of spontaneous flows, or nucleation sites for dynamical phase transitions.

The central concept for **topological textures in momentum space** is the Berry curvature. Due to their nontrivial topology, these materials can host topological surface, edge, hinge, and corner states [HK]. The topologically nontrivial character gives rise to unconventional transport properties, such as the quantum spin-Hall effect, the anomalous Hall effect, the chiral anomaly, and also to spin textures in reciprocal space. The relation between topological textures in real and momentum space has been studied only very little to date although they are closely linked [Hi]. The field is now expanding to include **non-Hermitian systems** where different phenomena arise due to the presence of exceptional degeneracies. In a non-Hermitian system, even a single band may be topologically nontrivial giving topologically stable vortices within the complex functional that plays partially the role of the energy [Be].

### 1.1.2. DESCRIPTION OF THE CHALLENGE (MAIN AIM)

The Action aims to address research questions related to topological textures from a broad perspective and in a **unified way.** This will require **coordinating the research efforts** across many physical systems and research communities in physics and mathematics. The universality of many textures needs to be made apparent, based on the concepts of topology. Since there is now a solid and extensive background in many systems, this is the right time to work towards the unification of the field. Research questions will be pursued with the general aims to :

- Develop a unified framework for topological condensed matter systems, including the identification of a suitable basis set of topological textures and the study of their robustness.
- Explore and support applications of topological textures, through studies of their nucleation and annihilation mechanisms combined with their interaction processes.
- Investigate textures and topologies in three dimensions, including the interaction between surface and bulk properties.
- Describe the dynamics of topological systems, including non-equilibrium processes, phase transitions, emergent phenomena, and the collective behaviour of crystals and fluids made of a large number of topological textures.

Each one of the research questions is linked to one of the working groups introduced in Section 4.

The unified approach will reveal general behaviours across systems and it thus has the potential to lead to fundamental results. The Action will then be able to address the challenge of understanding and manipulating **collective behaviour** of textures and emergent phenomena. Building on the answers to the fundamental research questions, the Action further aims to formulate proposals for topological textures and their dynamics to be tailored for applications in quantum and neuromorphic computing, for high density storage and efficient transport of information, and in advanced functional biomaterials.

Beyond previous studies where textures such as domain walls, vortices, or skyrmions were viewed as one- or two-dimensional structures, the challenge that lies ahead is to understand and describe their structure when they are embedded into **three or more dimensions**. The Action aims to describe the textures using the tools of topology in three dimensions. The loss of stratification leading to the introduction of flexibility of filaments (such as vortex filaments) will be studied as this enriches the properties (e.g., the dynamical modes) of the textures. The Action will furthermore address the effects of the surfaces on the statics and dynamics of topological textures, which is often the central question in cases where long-range forces are present.

In bulk materials (three dimensions), vortex rings or **Hopfions** that have proven to be relevant by theoretical predictions and by leaving their signature in experiments should now be fully studied. The topological features and the complexity added by the third dimension increase the experimental, theoretical, numerical, and mathematical difficulty. The challenge is to study the statics, the dynamics, and the features of Hopfions from a broad perspective. This will best be addressed by using a unified theoretical approach that applies to many of the relevant physical systems. The Action aims to work together across condensed matter systems, combining methodologies, drawing from mathematical theories and developing new mathematical and numerical tools.

## 1.2. PROGRESS BEYOND THE STATE OF THE ART

### 1.2.1. APPROACH TO THE CHALLENGE AND PROGRESS BEYOND THE STATE OF THE ART

Methods of theoretical (e.g., Hamiltonian, field-theoretic) and computational (e.g., spin dynamics, molecular dynamics) physics will be applied with two objectives. On the one hand, they will be necessary in order to make contact with experimental findings and, on the other hand, they will offer a solid and general enough description of the problems. Methods of applied (e.g., dynamical systems, theory of partial differential equations, variational theories) and computational (e.g., finite elements, structure-preserving algorithms) mathematics will be used and developed that have the potential to approach the physical problems from different perspectives and provide solutions by employing tools that have not been available to the community until now. Furthermore, new mathematical tools are likely to emerge that are required for the problems at hand. As experimental capabilities and methods vary significantly in different physical systems, the research projects will be adjusted accordingly. This means that experimental observations, (from network participants and from the literature) or potential observations (for example, of 3D textures), will define, in many cases, the scope of the research effort. The collaboration fostered by the present network will guarantee that research communities in physics and mathematics will be exposed to problems of significant potential with the aim of developing an interdisciplinary and complementary research effort. In short, methods, expertise and research culture from physics and mathematics will be combined in order to take steps for tackling fundamental problems in condensed matter systems.

The kind and the variety of the textures that are observed in different physical systems will set the stage for the various projects and will shape the collaborations between the individual research groups. Since the groups will work together in interconnected Working Groups (WG), as detailed in Sec. 4, flow of information will be enhanced. The networking activities will bring together experimental observations in different physical systems with strong theoretical and mathematical expertise. As a result, the Action aims to formulate **a unified approach** to seemingly disconnected observed textures and phenomena. The unified approach fostered by the Action and the full use of fundamental concepts will be instrumental in providing new and different insights compared to previous approaches that have been mostly subject-specific. This will be crucial for advancing topics of greater complexity, as is the case of 3D textures or collective behaviour. This section provides primary examples.

A study to identify, produce, and investigate **skyrmionic textures** of different symmetries and topologies, including topologically trivial ones is timely. The existence and stability of individual structures and collections of structures will be studied in various systems including those with long range interactions. A triangular **lattice** of skyrmions yields the apparently optimal configuration for chiral systems, such as chiral magnets, in analogy with the Abrikosov vortex lattice in superconductivity or BEC. It is an open question whether the triangular lattice is a global minimizer. The answer to this problem will help understand many recent experimental observations that show skyrmion lattice creation and annihilation due to varying parameters or by applying external fields. **Topological phase transitions** ideas, such as the Berezinskii-Kosterlitz-Thouless transition, should be extended to include complex topological textures beyond vortices or dislocations, and dynamical phase transitions seeded by topological textures. Within the same field, the nature of thermal and quantum melting transitions of crystals composed of topological defects is an open question. The framework of fractonic theories [Pr] can be used to explore various scenarios of disordering and melting of ordered states via proliferation of topological defects.

**The third dimension** will be taken into account where relevant. The structure of the textures in the interior of a film geometry will be studied in order to produce descriptions in terms of topological concepts extended to three dimensions. In the interior of films the concepts of domain walls, skyrmions, and vortices may be combined to give complex textures that can include singular points. Confined geometries will be studied. One of the aims will be to show their relevance in realistic systems as bits of information for storage and other applications.

In the theory of elliptic partial differential equations (common models in condensed matter), the extra dimension leads to questions about the relation of the symmetry of the equation to the symmetry of the solutions. For example, the symmetry of domain walls, when they are viewed as two- or three-dimensional objects, is related to the so-called **De Giorgi conjecture** for minimal surfaces [Sa]. The difficulty of the problem lies in the nonlinearity and nonlocality of the systems (e.g., due to the magnetostatic or electrostatic field) and in the nonconvexity of the variational models [Vi]. Modern techniques based on asymptotic analysis (using the method of Gamma-convergence) or calibrations will be applied.

**Topological soliton nucleation and annihilation** is a central issue with obvious practical importance, e.g., when solitons are used to encode and transfer information. The Action will address these processes starting from the mathematical results that clarify when such counter-intuitive processes are possible. In specific and subtle ways, topological constraints can be efficiently circumvented in order to obtain meaningful physical results. The role of the third dimension in such processes and the additional possibilities that it opens will be explored. Inspiration from the problem of singularity formation (or blow-up) will be sought and corresponding results and methods will be employed [Ta]. Furthermore, **topological soliton dynamics** (and dynamical modes) will be studied as it is often strikingly different from ordinary Newtonian dynamics. A significant tool comes from the mobility constraints of topological textures that can be naturally incorporated with the help of dualities towards tensor gauge theories coupled to fractonic matter [Pr]. Texture dynamics will be studied as a fundamental problem, while its understanding is also essential for making use of solitons in applications.

Understanding the nature of **dynamical transitions** induced by the proliferation and collective interactions of topological defects is an open challenge that the Action aims to address. Many of these systems with ordered ground states are driven out of equilibrium by external factors or even intrinsically driven by activity, as in biological systems. The formation and dynamics of topological defects is tightly connected to the underlying driven and dissipative dynamics of the system. Therefore, it is important to further develop a theoretical framework for better understanding the dynamical transitions to quantum turbulence in superfluids or active turbulence in active matter, their universality, and their connection to the collective dynamics of topological defects.

**Hopfions or vortex rings**, i.e., localised textures in 3D, will be studied as a main direction**.** In cold atoms and in nonlinear optical systems, these are described by extensions of the nonlinear Schrödinger (NLS) equation. In ferroelectrics, the appropriate models contain variables that pertain to polarisation as well as the lattice degrees of freedom in this system. This is expected to lead to phenomena beyond those found within NLS-type models. In magnetic systems, since the order parameter takes values on a sphere, a larger set of possible Hopfion configurations is expected. The dynamics of Hopfions, so far largely unexplored, will be studied as a separate topic as it is not a simple extension of the dynamics of lower-dimensional solitons.

**Interrelations of real and reciprocal space topology** will be explored using the concept of mixed Berry curvature [Su]. The goal is to use knowledge from real space topology (e.g. of the magnetic texture) in order to gain information about the reciprocal space topology, and vice versa, and to explore the limits of such a link. The Action aims at identifying systems and materials exhibiting nontrivial topology in both real and reciprocal space, e.g., Weyl semimetals or topological insulators with topologically nontrivial textures such as magnetic skyrmions [Pa], or magnet-superconductor hybrid systems hosting Majorana zero modes for quantum computation [Na]. Specific electronic, magnetic, and transport properties of these systems will be identified which result from the conjunction of nontrivial topology in real and reciprocal space.

For the simulation of models that are complex and nonlinear [Ab,Bao,Wa], the key challenge in numerical analysis is the design of numerical schemes that are **structure-preserving,** and can probe the enormous range of spatiotemporal scales associated with the formation and the evolution of topological structures. Numerical methods will be developed that are capable of preserving the properties (constraints on the solutions, energy laws) of the models at the discrete level. In addition, they should be constructed so as to be stable and convergent as the accuracy of the simulation increases.

### 1.2.2. OBJECTIVES

#### 1.2.2.1. Research Coordination Objectives

**O1. Coordinate research across physical systems and create a synergetic forum and platform** for researchers coming from different relevant physical systems. Training schools and workshops will be established to develop a common understanding of the subject matter across physics and mathematics. The proposers represent many physical systems. The Action aims to (a) introduce one or more additional systems in every workshop, (b) incorporate links between the physical systems in the training schools (documented with explicit connections in the lecture notes), (c) increase mobility of researchers to laboratories in areas different than their own (more than half of STSM mobilities), and (d) produce collaborative research work with researchers from different areas (more than 50 research papers).

**O2. Create an active link between the experimental/applied community and theoretical research.** The Action will expose theoretical (including numerical and mathematical) participants to problems that have a strong experimental or applied potential, where they can make a significant contribution. Vice versa, experimental and applied participants will profit from the already existing knowledge in the theoretical field. As a result, all communities will benefit and will be supported. WG2 will coordinate the contact with researchers in the industry, where a targeted effort is required. The Action aims for a significant number of new collaborations and corresponding publications between experimental/applied and theoretical researchers (involving approximately half of WG participants). The majority of the lecture courses in the training schools will cover specific solutions to problems related to applications by well-founded theoretical and mathematical methods.

#### 1.2.2.2. Capacity-building Objectives

**O3. Build a new community and achieve critical mass.** Most of the sub-communities working on topological textures in the respective physical systems are relatively small. The Action will combine these into a large interconnected and interdisciplinary community. This community is ideally placed to achieve breakthroughs by facilitating access to existing but inaccessible theoretical and mathematical knowledge and by fostering intense interaction between experimental and theoretical researchers from different disciplines. Events supported by the Action will, moreover, increase the visibility of research on topological structures. Success will be measured by the substantial number of participants (foreseen more than 50 participants in each main event and more than 15 participants in each WG event) and by the number of different fields of science and disciplines involved.

**O4. Foster the development of a joint research agenda.** The various successful research lines that are currently pursued by a number of smaller communities will be joined with the support of well-developed topology concepts. A joint research agenda for the field of topological textures in condensed matter will emerge. This will be reflected in the organisation of dedicated sessions in bigger conferences ( if possible in EPS, SIAM MS, JEMS, etc), the organisation of new conferences and workshops focused on synergies between communities (as, e.g., in the case of the TOPO conference).

**O5. Train the next generation of researchers offering comprehensive knowledge and methodological tools.** A generation of young physical scientists will be given the opportunity to gain a broad view of condensed matter beyond their specialisation, and to complement their intuition with a wide set of methodological tools. Also, a generation of mathematicians will be given the opportunity to exploit their technical achievements through their broader interpretation. The Action aims to train more than 150 young researchers and innovators (YRI) in training schools. Initiated by the training schools, YRIs from different areas and disciplines will have the opportunity to work together in an active research area that shows strong potential for fundamental results and for applications. More than two thirds of the STSMs will be devoted to YRI with more than half of them moving to a laboratory of a different community or discipline. The Action expects many YRI to produce research work pertaining to more than a single physical system together with researchers from different disciplines.

# 2. NETWORKING EXCELLENCE

## 2.1. ADDED VALUE OF NETWORKING IN S&T EXCELLENCE

### 2.1.1. ADDED VALUE IN RELATION TO EXISTING EFFORTS AT EUROPEAN AND/OR INTERNATIONAL LEVEL

A number of national networks on topological structures are in place, notably, "The skyrmion project" (U.K.), "SPP2137 Skyrmionics", Complexity and Topology in Quantum Matter (Germany), "Nanoskyrmionics" (Switzerland), and European, such as Quantum Flagship, TOCHA, Optical Topologic Logic, SKYTOP, 3D MAGiC, and others. They have gained a prominent place in international research and they demonstrate the timeliness of topological textures in condensed matter. The Action will complement these efforts by **connecting researchers who are currently members of separate communities** (ranging from soft matter to superfluids, solid state, and optics). It will also create a forum for the flow of information and ideas and the cross-fertilization between these disconnected communities. A wealth of experimental and applied work is currently available. The Action will strengthen, promote and enlarge this work by the strong involvement of **groups of theoretical researchers** in the wider community, which is expected to enhance both the fundamental and the applied side significantly. This network will be attractive for such researchers as it will offer a synergetic environment across physical systems. Furthermore, the Action will introduce a **group of mathematically oriented researchers** into the community. While there are some initial collaborations in place, the field doesn't yet benefit from potential links between mathematics- and physics-oriented approaches and research directions proceed fairly independently. During the Action, a research community within mathematics that has not yet achieved critical mass will become part of a wider community and benefit from the exchange of ideas, while they can readily contribute concepts, techniques, and results. As a final result the whole community will benefit, especially the **experimental laboratories** will be supported in the interpretation of their data and will get inspiration for new research ideas, ultimately leading to results with a wider impact.

Based on currently available reports, it is likely that exploitation of the results already available for certain physical systems can be extended and adapted to other physical systems. Notable examples include work on magnetic skyrmions that are relevant to ferroelectric or optical skyrmions, as well as vortex rings in optics and superfluids that are related to Hopfions in ferromagnets. While these systems differ in their physical properties and interactions, they are united by the underlying topological properties and concepts, including geometric phases, and homotopy (topological) transformations. A coordinated effort for **transfer of knowledge** from one research area to another is thus timely. The advantage obtained by combining knowledge across a connected and powerful community will lead to **European leadership** in the broad field of topological Physics**.**

The synergetic environment created within the Action will **attract new researchers** in the wider field of topological textures resulting in added value to current research efforts. This is because (a) Researchers who are working in isolated communities or on emerging topics will be encouraged to incorporate their results within a wider topic and thus broaden their impact; and (b) Researchers who are interested in exploring topological textures in new contexts will find a community where tentative or high-risk ideas can be discussed and tested, potentially leading to their productive incorporation in the field of topological physics. The circulation of ideas in the wide context described clearly increases the potential for a breakthrough in relation to existing efforts. The Action workshops will be the main forums where researchers can connect with the network community while promising collaborations will be supported by STSMs. As an essential first step, Action workshops will be broadly advertised, e.g., by communications in the main conferences of the communities that the Action will target. The added value of this process will be reflected in (and measured by) the active participation of new researchers in the Action and in the additional expertise and concepts that they will bring to the field.

The Action includes a broad mix of expertise, in physics and mathematics, that is suitable in order to develop **training opportunities** of a sort not usually available. The grown skills will encompass applications (e.g., in spintronics, optics, liquid crystals) and rigorous methods (e.g., Hamiltonian, variational) while the Action will also allow for a flow of information between them. The Training Schools will offer **a balanced mix of fundamental and applied knowledge,** attractive to the theoretical, applied, and experimental communities. This will be reflected in the mix of participants. Synergies with existing schools will be sought in cases of mutual benefit.

## 2.2. ADDED VALUE OF NETWORKING IN IMPACT

### 2.2.1. SECURING THE CRITICAL MASS, EXPERTISE AND GEOGRAPHICAL BALANCE WITHIN THE COST MEMBERS AND BEYOND

Most of the research communities involved in the Action, including topological research in magnetics, ferroelectrics, quantum systems such as BECs and topological band theory, soft matter such as liquid crystals, optics, and mathematics (as mentioned in Section 1.1.1), are relatively small. Each of these areas is represented by several researchers across more than 20 countries (more than 60% ITCs) in the group of proposers for this Action, thus ensuring a cohesive emerging community, comprising well in excess of 200 researchers and thus **achieving critical mass.** The proposers also include a company on scientific computing, a metrology agency, and an organization that supports women in STEM and communicates science. As the Action aims to build a new research community, this network has the opportunity and obligation to generate an inclusive welcoming environment for researchers of different nationalities, gender, sexual orientation, geographical background and research experience. A high percentage (more than 40%) of women is included in the proposers, close to parity and well above the average in the area of the Action, and 45% of the proposers are YRI.

Importantly, separate communities are **brought together in a coherent way:** they share methods, concepts, results, or objectives. The experimental and theoretical researchers are working across a range of different physical systems but they are related by similar geometry (topology). The mathematical researchers are working on the same physical systems as the physical scientists albeit from different points of view or by using different and complementary tools. The **thematic and disciplinary diversity** of proposers is an advantage which will act as a seed for attracting more researchers and groups to join the Action. The network offers all participating researchers an open avenue to benefit from the joint knowledge base in topological structures across different disciplines, and in turn to promote their individual contributions across the wider community of condensed matter and beyond for achieving enhanced impact.

The group of proposers offers **wide-ranging expertise** for achieving the objectives, in experimental techniques (including imaging and time-resolved measurements numerical (including numerical analysis methods such as finite element methods, scientific computing on local, non-local systems and on multi-scale systems), and theoretical or mathematical (including Hamiltonian and nonlinear dynamics, statistical physics, integrable classical and quantum systems, multi-scale modelling, differential geometry, algebraic topology, calculus of variations, and partial differential equations) methods. Many among the proposers have experience in organising research and training events that have brought together participants from different disciplines on topological physics. In addition, some of the proposers have carried out research across two or more topics or disciplines in physical systems related to this Action. This network is thus in an advantageous position in order to start working in a synergetic environment and achieve the objectives.

The Action will work in order to **attract researchers from fields** where the ideas and concepts of the Action community can have a positive impact or can be developed further or in different ways. This can be, for example, soft matter systems including patterns in polymer-based systems, related astrophysical systems/matter, emerging areas such as non-Hermitian systems, and also chemical, or various biological systems. The community is thus expected to grow during the lifetime of the Action, as will be measured by number and diversity (thematic, disciplinary, geographic, etc) of participants in its activities. Some of the research areas (physical systems) included in the Action are developed only in certain countries while only isolated researchers work in others. In order to address this issue, the Action will support the **transfer of expertise** in specific physical systems across the participating countries by training schools, STSMs, and virtual mobility. For example, researchers from magnetic and optical systems in bigger countries (in west Europe) can support groups in magnetic and optical systems and also in BEC (a related area) who are based in participating ITCs to foster new activities. Longer STSMs in both directions will be organised, especially in cases where there is complementary expertise in different physical systems across two or more geographical areas, so that all places will benefit by incoming expertise.

Building the **critical mass while maintaining geographical balance** will also be reflected in that the network will be able to (a) widen the scope of the Action and attract researchers from a continuously increasing number of condensed matter topics and geographical areas, (b) make their methods and results wider known also outside condensed matter (e.g., in high-energy, atomic physics, chemistry, biology), for example, by Action participants getting invited to respective meetings for the presentation of the Action results (approximately ten such invitations per year are expected as some of the participants have already links outside condensed matter), (c) support young researchers and innovators, coming from experimental, theoretical physics, or mathematics who will be suitably trained and supported so as to pursue a career in topological textures. The Action will run a targeted effort (led by WG4) in order to approach more researchers from ITCs and also include NNCs who have the necessary background (and may or may not be yet directly related to topological physics). It will offer them the opportunity to participate in STSMs and Training Schools. The virtual mobility scheme will be used for initiating contacts, and also whenever physical mobility is not convenient, especially in the case of NNCs.

The communication within the network and also between the network participants and wider academia or other stakeholders will be assisted by virtual meetings and communication methods, providing a cost-effective and convenient way to connect the large number of anticipated participants. Virtual networking will be used for some WG meetings and for online seminars, in order to support whenever possible the hybrid format of Action workshops, and for complementing Training Schools with a blended learning component. Furthermore, virtual mobility will be used as a general tool but also specifically for supporting the contact with NNCs and ITCs , that is for the involvement of more YRIs from these countries. The best combination of virtual or hybrid networking tools will be one of the significant subjects of deliberation for the Action MC. The implementation of the networking tools is detailed in Sec. 4.

### 2.2.2. INVOLVEMENT OF STAKEHOLDERS

Our immediate stakeholders include **experimental laboratories** in academia, industry and government agencies (such as metrological, included in the group of proposers) who can utilize the developed concepts and unified approach in order to better model existing effects and to develop novel applications. Especially experimental laboratories in ITCs will acquire access to expertise across a wide spectrum of physical systems and they will be able to develop those that are most promising for their own capabilities. Laboratories will be able to exchange methods and expertise developed in various contexts. Key researchers will be invited to Action workshops, and STSMs with their laboratories will follow whenever possible.

The Action output will bring benefits to **academic communities in related disciplines**. The Action will seek interaction with scientists in nucleonics, elementary particles, atomic physics, and also chemistry and biology, making use of existing contacts of researchers within the network. The Action intends to invite individual researchers beyond the network to deliver outline talks to our community and in turn strive so that researchers from within the network make presentations at conferences organised within those research communities.

The network will extend activities, building on existing collaborations, to **industrial partners and government bodies** in order to promote the results. There is no shortage of applications for complex topological systems in condensed matter: Industries on magnetics are using topological solitons, such as vortices, for sensors while skyrmions have been proposed for information storage and computation. Optical vortex lenses are used for nano-lithography, laser cutting, data transfer, and optical tweezers manipulations. Localised topological solitons (because they can be as small as a few nanometers in size) are proposed for the manipulation of qubits with generated optical or other fields. All our workshops will include a dedicated session aimed at linking the Action results to existing or proposed applications, and relevant industrial stakeholders will be specifically targeted with invitations. Given the Action composition, it should be clear that the overall purpose is to take only the first steps towards industry involvement. Extensive industry involvement is not within the design and capabilities of this project. These activities will be coordinated by WG2 as detailed in Section 4.

The industry on **scientific computing** (applied to natural sciences, engineering, general education, etc) can potentially use the framework developed as part of this Action and incorporate it in their relevant products (this includes a proposer). The computing community is expected to benefit from methods of high mathematical complexity that require tools from applied mathematics including state-of-the-art numerical analysis methods.

Beyond contributions of industry members to the Action workshops, STSMs to and from the industry will be realised. A workshop in the form of a **retreat** will be organised where members of the Action and members of the industry will meet in order to find overlapping objectives and define common projects. A task group will be formed in order to pursue this task, including a participating company who are already supporting many industrial players.

National and European policymakers will be contacted and invited to meetings of the MC to communicate the potential of topological textures as candidates for industrial applications and as an educational subject. Topology is a highly attractive and visually inspiring topic that catches the imagination of the general public, children, and students alike. This will be an excellent example to promote interdisciplinarity in primary and higher education. At the university level, students in natural sciences and in mathematical sciences can receive overlapping education and develop a common language that would allow them to work together in academia and the industry. The Action will demonstrate how this can work in its area of interest (e.g., in the Training Schools) so that policy makers will have a solid argument to support the effort.

# 3. IMPACT

## 3.1. IMPACT TO SCIENCE, SOCIETY AND COMPETITIVENESS, AND POTENTIAL FOR INNOVATION/BREAKTHROUGHS

### 3.1.1. SCIENTIFIC, TECHNOLOGICAL, AND/OR SOCIOECONOMIC IMPACTS (INCLUDING POTENTIAL INNOVATIONS AND/OR BREAKTHROUGHS)

*Scientific impact.*

Based on its broad expertise, the Action will be able to address a wide spectrum of problems related to topological textures. As similar textures may appear in a large number of physical systems, **results will affect many research areas**. All subcommunities will be supported by the developed theoretical methods and tools. The expertise and results achieved within specific systems will be transferred across the Action and will affect research in all other physical systems in a straightforward way. The successful cross-fertilization of condensed matter systems holds the potential for innovations in a series of physical systems.

Beyond condensed matter, analogous topological textures exist also in areas such as elementary particles, chemical, and biological systems, etc. The broad classification of topological textures that will be obtained will be a source of inspiration and a useful base for communities also **outside condensed matter**. The methods that will be proposed and explored regarding generation and annihilation of topological textures can be the starting point for new ideas, especially in various applied communities.

The research effort on **3D structures** is confronted with fundamentally unexplored problems. The approach of the topic in a unified way will allow the Action to take a leading role in the research area and in the wider community as these problems will be taken up in the near future. Many results produced within the Action will be expressed by exploiting constructive mathematical abstraction, thus they will be suitable for exploitation also in other disciplines. Once fundamental results on the statics and dynamics of 3D textures will be achieved, it can initiate a broader exploitation of the idea of 3D topological structures in the physics community and also outside it. The study of a little explored topic such as 3D topological textures from a combined physical and mathematical perspective clearly holds potential for scientific breakthroughs.

*Technological and Economic impact.*

Topological textures can combine memory and logic functionality and they are candidates for devices beyond CMOS, which can work as hardware (enabling in-material computing) for performing **neuromorphic computations** [So]. The non-linear dynamics of these textures can prove instrumental in solving complex neuromorphic tasks which require the inputs to be projected in a high-dimensional space. Systems with long-range interactions and 2D or 3D structures can potentially mimic a large number of synaptic connections in neural networks, i.e., a task which is impossible to implement with the current state-of-art CMOS devices. The Action output can lead to optimization of the functional behaviour of topological textures for the available neural network architectures/algorithms. The aim is a great improvement of energy efficiency in the resulting technologies.

Skyrmions nucleated inside a Magnetic Tunnel Junction can be used to represent nanoscale bits for dense storage. In addition, they can be used to perform logic operations exploiting their interactions via magnetostatic and exchange forces. This opens up a unique possibility to propose CMOS-compatible hybrid hardware for **logic-in-memory computing** based on skyrmions. Such logic-in-memory architectures can lead to a significant technological breakthrough as they have a huge advantage in energy efficiency and speed over the conventional Von Neumann architectures with physically separate storage and logic units (which require extensive interconnects leading to huge energy dissipation). Analogous ideas involving topological defects in liquid crystals lead to the implementation of **logic gates** and, further, to classical digital and nonclassical continuous computation strategies [KD].

Topological structures may be parts of innovative qubit architectures. An example are magnetic skyrmions of a certain morphology, for which the ability to induce Majorana bound states is predicted [Re]. The Action output can contribute to the development of innovative technologies in the field of quantum computing including **topological quantum computation**, fitting into the scope of the European Quantum Flagship. Another example is the helicity of a skyrmion texture which could become a quantum degree of freedom in quantum logic elements [PP].

A theoretical description of topological defects in active matter will offer a versatile set of tools to bridge the gap between biological and synthetic materials with tailored properties and functionality. This convergence has the potential to address some of the current challenges in **biotechnology** and **regenerative medicine** in terms of groundbreaking advances in regenerative tissue engineering, microfluidics and controlled drug delivery and stem cell biotechnologies.

The planned retreat-type workshop with industrial stakeholders will offer the possibility to prioritise the above directions and work on further collaboration with respective industries.

## 3.2. MEASURES TO MAXIMISE IMPACT

### 3.2.1. KNOWLEDGE CREATION, TRANSFER OF KNOWLEDGE AND CAREER DEVELOPMENT

Knowledge will be created within an interconnected community of physics, computational science, and mathematics. This will be based on a number of methods contributed from the different disciplines working synergistically. **Paradigms of topological textures** that have not been known or have not been fully appreciated yet, for example in quasi-2D or 3D settings, are expected. The end result will be a reasonably complete theory of the properties of complex topological textures and their experimental demonstration and exploration.

The Action will seek to transfer the created knowledge to a **community as wide as possible**. To this end, the Action will (a) extend the network by attracting researchers from further condensed matter systems, (b) identify further areas and systems where the notion of topological textures is or can be applicable and involve the corresponding scientists in the developing research community.

Knowledge will be transferred to groups in less research-intensive regions, particularly in **ITCs and NNCs**. This will initially be focused on theoretical groups that will directly benefit from the interaction with a wider theoretical community. The Action events will also seek contact with experimental laboratories from the above countries and support them by exchanges and training.

Transfer of knowledge will also be toward **experimental groups** that are setting up demanding experimental programs and they certainly need to base them on challenging and well-founded questions (as in Sec. 1.2.1). Some questions, that are often confined to be discussed among theoretical groups, for example, fiber bundles in topology or formulations for topological soliton dynamics, will be developed and exploited in a more complete way with the contribution of experimental laboratories. For this purpose, the Action will seek to organise, if possible, specialized symposia within application-oriented large events.

Partners and end users who can potentially incorporate the complexity of the Action results in the design of products will be approached via the **Technology Transfer Offices (TTOs)** of the participating institutions. The TTOs will be responsible for the identification and protection of the generated IP and will handle the patent applications that are expected to be filed during the Action. Beyond the regular network activities, the industrial stakeholders will be invited to interactive seminars organised by the TTOs. Seminars that will be shared by many Action nodes will be organised, so that their impact will be enhanced and it becomes highly probable that promising contacts between Action participants and an industrial partner are eventually established.

The creation of a multidisciplinary community around a broad subject in condensed matter will create a forum for the **development of the career path** of YRI by exposing them to problems that are shared by a wide but previously disconnected community. Participating research groups will develop projects for PhD positions involving secondments between them so that research students will acquire training by more than one group (in the spirit of ITN networks). STSMs as well as virtual mobility will be used for this purpose. Postdoctoral researchers will work in an interdisciplinary environment securing privileged positions in the scientific community of the future. Their participation in interdisciplinary Working Groups and in the Training Schools will be instrumental towards this end. In the Action workshops, there will be presentations by the organization for women in STEP (task of WG4) and by industrial partners (task of WG2) so that YRIs will get informed and get exposed to opportunities for a career also outside academia. Finally, the participating senior researchers will have the opportunity to expand the scope of their research in a field as wide as possible, thus achieving higher status in the community.

### 3.2.2. PLAN FOR DISSEMINATION AND/OR EXPLOITATION AND DIALOGUE WITH THE GENERAL PUBLIC OR POLICY

**The webpage of the Action** will be the standard tool for dissemination and communication activities for the participants. it will contain the activities of the action, the work and expertise within the WGs and the events that will be organised by the Action or those related to the Action. Furthermore, social media accounts will be established (*X* (*Twitter)* and *LinkedIn* for academic, *Instagram* and *YouTube* for the public, possibly other as social media evolve). They will be used to post the Action regular work and make announcements for news and events. The Action webpage will be fully public and linked with the social media accounts.

**Training Schools** will be organised (one each year) and targeting more than 40 YRI in each school. The on-site training will be preceded by online seminars and courses (part of the "Seminar" activity) thus creating a blended training environment. **Training activities** will occupy a dedicated part of the webpage. Following the training schools, the webpage will be enriched with material for the completed courses and tools for interactive online teaching. In this way, a much wider audience of research students, YRI, and early career professionals will be reached. The Training Schools format (mix of participants, courses format, length), if possible, will be designed to be compatible with the *Blended Intensive Programmes* of Erasmus+ so that schools could be continued after the end of the Action. Courses and teaching materials will be tested and incorporated in graduate programs of participating institutes.

Dedicated **Symposia** will be organised, if possible, by Action participants within large events, with the purpose of increasing the visibility of the Action (possibly two symposia per year, but only a few of them will be partially funded by the Action). Furthermore, YRI from ITC countries will attend high profile events and they will thus not only benefit as individuals but they will also disseminate more widely the Action results (the Action envisages possibly to support five YRI per year).

**Publications** in scientific journals will be the main tool for the dissemination of scientific results to the broader scientific community. Open-access journals will be targeted (for most of the publications) when these are of high-quality. All papers will also be published on the arXiv, as is common practice in our constitutive communities. Research data will be treated in compliance with the FAIR (Findable, Accessible, Interoperable and Reusable) principles. The main codes and the produced data will be made available via repositories such as Github and they will be linked via the Action website.

The strategy for an efficient approach and **dialogue with the general public** is inspired by the fact that structures with a nontrivial topology can be visually attractive and challenging. For example, knots and other particularly complex patterns of topological nature have been used in pieces of art. Newspaper articles are using topological textures to attract readership, as in a recent article [DS]. The Action will set up an account in social media (e.g., Instagram) where the most attractive pictures and figures from the network studies will be posted regularly.

Based on the pictures and figures produced within the network, **a video** will be produced that will convey the main ideas of topological textures in a way suitable for a wide audience (working title "Why knot add complexity in life?"). This will be mainly based on geometrical ideas that are inherent in topology while we all are familiar with these from our basic education. Figures, pictures, geometry, and fundamental ideas will be put together in **a booklet**. The video and booklet will be available in the Action website and social media accounts and they will be used during the activities with the public.

**The general public of all ages** will be approached with dedicated events (for example, talks in TeDx events) and actions embedded in outreach activities (e.g., researchers' nights, science festivals) organised in participating institutions or other organisations. **For youngsters of all ages** and educators, the Action will make corresponding presentations in summer schools for school students and open days for prospective students at universities in participating countries. Given the expanding participation of schools in the ERASMUS+ program, the Action will seek synergies if possible, i.e., offering if possible some of the above events to be embedded in ERASMUS+ projects for secondary education.

The contact with students will be based on an interactive approach via videos, computer games, competitions, etc. For this purpose, the Action will seek to **build on already successful practices and events**. Specifically, an activity similar to MATh-en-JEANS (France) can be introduced to selected other countries and include the entire scientific scope of the network. This activity develops twinning actions between a scientist and highschools. The aim is, via posing a research problem, to build a different image of mathematics than that of a strict scientific field. Here, a scientist could propose a problem such as domain walls; they are governed by the eikonal equation whose solutions are distance functions and the ridges (singularities) can be viewed as simulating sand castles.

For university undergraduate students, the Action will set up groups of interested students (encouraging the participation of women). A supervisor will create a collaborative environment by giving attractive problems related to topological textures, beyond the standard curriculum. The short-term aim will be to prepare students for participation in olympiads, while the long-term aim will be to prepare them for research work. Groups in different nodes will work together in virtual format and they will finally hold a short physical meeting.

# 4. IMPLEMENTATION

## 4.1. COHERENCE AND EFFECTIVENESS OF THE WORK PLAN

### 4.1.1. DESCRIPTION OF WORKING GROUPS, TASKS AND ACTIVITIES

A **mix of experts** from different condensed matter systems and disciplines will meaningfully work in each working group (WG) towards common goals. Multiple research questions are shared between WGs and the tasks of each one constitute crucial input for the work of at least one of the others. WGs will also take over tasks of training activity, and contact with other scientific communities, stakeholders, and the public. With this structure, the Action will work as a whole and it will achieve cross-disciplinary knowledge transfer.

A regular **online seminar** between the network participants has already been in place. It will expand to include interactive seminars with industrial stakeholders organised by the Knowledge Transfer Offices. It will also serve as preparation for the Training Schools, so that a blended training program is formed. WG meetings will be organised (some of them as hybrid mini workshops) and they will work in a complementary way with the seminar. This structure will secure a continuous interaction between the Action participants.

The Action will secure a high level of involvement of ITCs throughout the life of the Action. It will involve YRIs and respect gender balance in all aspects of the implementation including in Action leadership positions. This will be achieved through specific tasks of WGs and activities as explained in the WG descriptions.

**WG1. Topology and transfer of expertise**

The group will focus on identifying and describing topological textures in a broad sense, both in real and in momentum space. Using mathematical expertise, they will develop a unified description beyond the state-of-the-art. The group will investigate topological textures in realistic nonlinear models, in 1D, 2D, and quasi-2D geometries. They will explore the connections of topological textures between different systems, and seek to reveal their versatility and significance in Nature. This group will lead the dissemination activities and the effort to expand the network.

Tasks.
T1.1. Identify and study topological textures, and demonstrate their ubiquity and versatility.
T1.2. Find analytical and numerical solutions of models. Demonstrate observations of textures.
T1.3. Demonstrate the link between real space and momentum space topological textures.
T1.4. Expand the network, in particular reaching out to YRI.
T1.5. Coordinate dissemination activities.

Activities. A1.1. Organise Workshop “Polytopo-Textures”. A1.2. Organise Training School on “Ubiquity of topological textures in condensed matter". A1.3. Set up and maintain the Action webpage and social media accounts for research. A1.4. Organise Symposia within large events. Contact groups within and beyond condensed matter.

**WG2. Topological textures and applications**

The group will focus on the issue of the generation (nucleation) of topological textures as well as in methods for their controlled annihilation (deletion). They will study texture manipulation including the role of quasi-2D geometries and they will formulate proposals for applications. This group will lead the communication with industrial partners and policy makers.

Tasks.
T2.1. Identify and classify methods for nucleation and annihilation of textures.
T2.2. Simulate numerically nucleation/annihilation processes and produce realistic results. Demonstrate related observations.
T2.3. Formulate proposals for manipulation of textures related to applications (computing, biotechnology, and other).
T2.4. Contact industry, and policy makers.

Activities. A2.1. Organise Workshop "Polytopo-Generate". A2.2. Organise Training School on "Nucleation and annihilation of topological textures: Theory and realisations". A2.3. Organise retreat-type meeting with industry. A2.4. Coordinate exchanges between academia and industry. A2.5. Produce white papers for policymakers.

**WG3. Textures in three dimensions**

The aim is to give a mathematical formulation of little-known textures localized in three dimensions, and describe different textures across condensed matter in a unified way. The relation of quasi-2D textures with localised textures in the bulk will be studied. The group will lead the outreach activities.

Tasks.
T3.1. Study topological invariants in three dimensions and establish links to physical systems.
T3.2. Identify, describe, and simulate numerically textures in three dimensions in condensed matter. Demonstrate observations of textures.
T3.3. Formulate the links between 3D textures and quasi-2D systems.
T3.4. Contact the general public.

Activities. A3.1. Organise Workshop "Polytopo-in-3D". A3.2. Organise Training School on "Topological textures in the bulk". A3.3. Produce booklet and video for outreach, establish presence in social media for the public. A3.4. Organise talks and events for the public ( e.g. Researchers' nights, TeDx, MATh-en-JEANS) and report on best practices.

**WG4. Dynamics and Topology.**

The group will study the solitary wave and particle-like character of topological textures, and collections of textures. They will study non-equilibrium processes and phase transitions. The group will lead the efforts to include underrepresented groups, and more ITC and NNC researchers in the Action. Also, they will pave the way for the network sustainability.

Tasks.
T4.1. Develop formulation for dynamical behaviour in a unified way and link with topology.
T4.2. Study, simulate, and classify response of topological textures to external forces. Demonstrate observations.
T4.3. Study collective motion of topological textures and their connection to non-equilibrium dynamical transitions.
T4.4. Support research and innovation work in ITCs and NNCs. Encourage talent in underrepresented groups.
T4.5. Identify opportunities for future collaboration and elaborate sustainability strategies.

Activities. A4.1. Organise Workshop "Polytopo-Dynamics". A4.2. Organise Training School on "Dynamics of topological textures in condensed matter". A4.3. Identify researchers and coordinate exchanges (STSMs and virtual) that include ITCs, NNCs, and underrepresented groups. A4.4. Produce a roadmap document for the Action topic. A4.5. Produce a report describing sustainability strategies.

### 4.1.2. DESCRIPTION OF DELIVERABLES AND TIMEFRAME

Regular activities such as Workshop and Training School organisation, MC meetings, and STSMs, will be documented in the progress report (**PR2,** in month 24) and Final Achievement Report (**FAR**) that are mandatory. In addition, a report following each Workshop and each Training school will be delivered along with further deliverables, as detailed here. The deliverables are also shown on the Gantt chart.

**D1.** Launch of webpage of Action, and social media accounts. (Month 3)

**D2.** Report, following the Workshop, on (a) Workshop activity and webpage, (b) WGs work and achievements, (c) expansion of topics and participating researchers in the Action, (d) joint works across topics or disciplines, (e) geographical expansion to ITCs and NNCs, involvement of YRIs, and gender balance, (f) results of potential interest to stakeholders, (g) related presentations of the Action participants to third party events within and outside condensed matter, (h) dissemination activities. (Month 10)

**D3.** Report, following the Training School, including teaching material, i.e., (a) school program, participation and webpage, (b) summary of lecture notes, (c) training material (exercises, numerical codes), and (d) multimedia material (processed videos of some lectures). The report will also includes (e) program of the regular seminar, and (f) mobilities of YRIs. (Month 10)

**D4.** Booklet and video animation for the public. (Month 15)

**D5.** Report as in D2. (Month 22).

**D6.** Report as in D3. (Month 22)

**D7.** Report on retreat-type workshop and white paper for policymakers on topological textures for industrial applications and as an educational subject. (Month 25).

**D8.** Report as in D2. (Month 34).

**D9.** Report as in D3. (Month 34)

**D10.** Report describing sustainability strategies. It should contain a roadmap for the scientific directions in topological textures across condensed matter and of the possibilities to sustain the network. (Month 37)

**D11.** Report on presentation for outreach including (a) presentation material, (b) video of talks, (c) program and participation data of events organised. (Month 40)

**D12.** Report as in D2. (Month 46).

**D13.** Report as in D3. (Month 46)

### 4.1.3. RISK ANALYSIS AND CONTINGENCY PLANS

*Risk.* (Coordination) Given the relatively wide scope of the topic and the involvement of researchers from different disciplines, disagreements may arise between the partners regarding the priorities of the network as the scientific topic will be evolving over the lifetime of the network.
*Plan.* Priorities will be discussed during MC meetings and possible disagreements will be identified and resolved at an early stage. In particular cases a Core Group may be formed in order to formulate suggestions to the MC. The guiding principles will be the MoU of the Action and an assessment of the currently most promising directions. In the unusual cases that no consensus is reached, decisions will be taken by simple majority.

*Risk.* (Community) The Action aims to operate many meetings as hybrid events in line with post-covid19 trends. The number of remote WG participants is too high compared to on-site participants.
*Plan.* On-site participation in workshops will be encouraged by offering invitations to on-site participants. Some WG meetings can be held entirely online so that, consequently, a rule of on-site participation could be imposed to other meetings.

*Risk.* (Community) Low intra- and inter-WG networking activity.
*Plan.* Increase the number of STSMs (including those for PhD students) between research groups of different topics or disciplines. Use COST funding for open-access expenses for articles with 3+ members from different groups.

*Risk.* (Impact) Low visibility of the network.
*Plan.* Increase conference grants. Invite key researchers from outside the network, to workshops (on-site) and to WG meetings (possibly virtual). Organise symposia by groups of Action participants in large conferences.

*Risk.* (Capacity) Limited flow of expertise between the geographical areas, especially towards ITCs.
*Plan.* Virtual Mobility tools will be used in order to involve more participants from identified areas. Special schemes will be used in order to support ITC researchers: dedicated STSMs as follow-ups of virtual mobility, dedicated STSMs to ITCs, conference grants, WG meetings with thematic combining expertise of ITC researchers with new topics.

### 4.1.4. GANTT DIAGRAM

| | Year 1 | | | | Year 2 | | | | Year 3 | | | | Year 4 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| activity/quarter | Q1 | Q2 | Q3 | Q4 | Q1 | Q2 | Q3 | Q4 | Q1 | Q2 | Q3 | Q4 | Q1 | Q2 | Q3 | Q4 |
| MC meeting | kick-off | | | PR1 | | | | PR2 | | | | | | | | FAR |
| Webpage / Social media | D1 | | | | | | | | | | | | | | | |
| Regular Seminar | | | | | | | | | | | | | | | | |
| WG meeting | kick-off | | | | | | | | | | | | D10 | | | |
| Workshop | | | | D2 | | | | D5 | | | | D8 | | | | D12 |
| Training School | | | | D3 | | | | D6 | | | | D9 | | | | D13 |
| STSM | | | | | | | | | | | | | | | | |
| Outreach / policymakers | | | | | D4 | | | | D7 | | | | D11 | | | |

The Regular Seminar will include (a) regular research talks, (b) lectures series in preparation of Training Schools, and (c) interactive seminars from industrial stakeholders invited by the Transfer of Knowledge Offices.

Outreach activities will precede the corresponding deliverables.

WG workshops will include research work and work towards deliverables.

PR1, PR2, FAR are the mandatory reports.

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